Chapter 1.

Use of formulae.

Situation One

A person has a modern cookbook that gives oven temperatures in degrees Celsius but an old oven with the dial graduated in degrees Fahrenheit.

The person wishes to convert some Celsius temperatures in the book to Fahrenheit temperatures in order to correctly set the oven dial.

A book she has tells her that to change a Celsius temperature ($°C$) to the equivalent Fahrenheit temperature ($°F$) she can use the following rule, or **formula** (plural: formulae or formulas):

$$F = \frac{9}{5}C + 32.$$

Change the following Celsius temperatures to Fahrenheit.

(a) 200°C (b) 170°C (c) 150°C (d) 100°C

Situation Two

For some medical procedures, for example administration of chemotherapy, the surface area of the patient's body may need to be known. This is not an easy thing to measure so instead there are various formulae that can be used. One such formula, known as the Mosteller formula, provides an estimate of the surface area of the patient, A m^2, using the weight of the patient, Wkg, and the height of the patient, h cm.

The Mosteller formula: $$A = \sqrt{\frac{W \times h}{3600}}$$

According to this formula estimate the surface area of a patient with:

(a) Weight 80 kg, Height 169 cm. (b) Weight 61 kg, Height 155 cm.

(c) Weight 79 kg, Height 183 cm. (d) Weight 70 kg, Height 159 cm.

Use of formulae.

On the previous page, the formula

$$F = \frac{9}{5}C + 32$$

allowed values of F to be determined for given values of C, and the formula

$$A = \frac{W \times h}{3600}$$

allowed values of A to be determined for given values of W and h.

The formulae on the previous page could have been stated in words rather than symbols. For example $F = \frac{9}{5}C + 32$ could have been stated as follows:

> To convert a Celsius temperature to the equivalent Fahrenheit temperature multiply the Celsius temperature by nine, divide by five and then add thirty-two to your answer.

Clearly the statement $F = \frac{9}{5}C + 32$ is more concise but it does require us to be able to understand this use of letters as a mathematical shorthand.

How did you use the formula to work out the Fahrenheit equivalent of 200°C?

- Perhaps you substituted $C = 200$ into the formula:

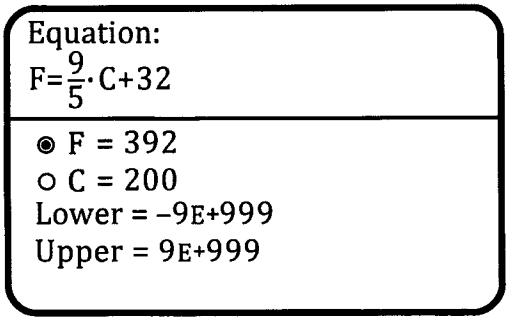
  $$F = \frac{9}{5}C + 32$$

  If $C = 200$

  $$F = \frac{9}{5}(200) + 32$$
  $$= 360 + 32$$
  $$= 392$$

  Note that in this method we need to remember the "rule of order". The 200 needs to be multiplied by 9 and divided by 5 **before** we add 32.

- Perhaps you determined the values of F for the various values of C using the ability of some calculators to determine unknown values in a formula given sufficient information.

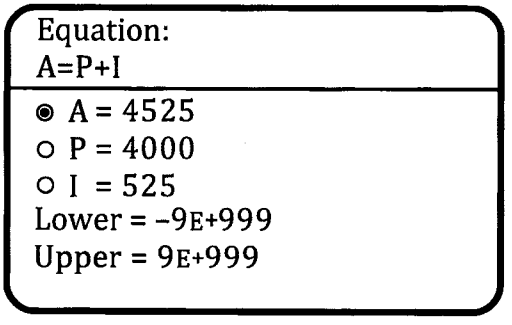


Example 1

Formula: $\boxed{A = P + I}$. Find A given that $P = 4\,000$ and $I = 525$.

Start with the given formula: $A = P + I$
Substitute in the known values: $A = 4\,000 + 525$
Evaluate: $= 4\,525$
Thus $A = 4\,525$.

Note: The formula given in the above example is very straightforward so the above procedure would probably be carried out mentally. However, as before, the same result can be obtained using the ability of some calculators to determine unknown values in a formula given sufficient information.



Example 2

Formula: $\boxed{s = ut + \frac{1}{2}at^2}$. Find s given that $u = 5$, $t = 3$ and $a = 4$.

Start with the given formula: $s = ut + \frac{1}{2}at^2$
Substitute in the known values: $s = (5)(3) + \frac{1}{2}(4)\,(3)^2$
Evaluate: $= 15 + \frac{1}{2}(4)\,(9)$
$= 33$

Thus, when $u = 5$, $t = 3$ and $a = 4$, $s = 33$.

Alternatively use the appropriate facility of some calculators.

Example 3

Formula: $A = 250\,(1{\cdot}08)^n$

Find A correct to one decimal place given that $n = 14$.

Either by using a "solve facility":

Equation:
A=250·1.08^n

◉ A = 734.298406064415
○ n = 14
Lower = -9E+999
Upper = 9E+999

or "by hand":

$$A = 250\,(1{\cdot}08)^n$$
$$\therefore \quad A = 250\,(1{\cdot}08)^{14}$$
$$\approx 734{\cdot}298$$

Thus $A = 734{\cdot}3$ correct to one decimal place.

When $n = 14$, $A = 734{\cdot}3$ correct to 1 decimal place.

In the next example only the "by hand" method is shown. If you wish to use the "solve facility" of your calculator instead do still read through the example to check that you understand the process of substituting values into the formula and then check you can obtain the same answer using your calculator.

Example 4

A particular industrial process involves the mining and production of a particular metal to various levels of purity. The production of 1 tonne of the metal of p% purity costs the company $\$C$ where a good approximation of C is given by the formula:

$$C = 1\,500 + \frac{125\,000}{100 - p}$$

Find (a) the cost of producing one tonne of the metal of 90% purity,
(b) the cost of producing one tonne of the metal of 99% purity.

(a)
$$C = 1\,500 + \frac{125\,000}{100 - p}$$

We are given that $p = 90$, thus
$$C = 1\,500 + \frac{125\,000}{100 - 90}$$
$$= 1\,500 + \frac{125\,000}{10}$$
$$= 14\,000$$

To produce one tonne of the metal of 90% purity costs approximately \$14 000.

(b) $$C = 1500 + \frac{125000}{100 - p}$$

We are given that $p = 99$, thus $$C = 1500 + \frac{125000}{100 - 99}$$

$$= 1500 + \frac{125000}{1}$$

$$= 126500$$

To produce one tonne of the metal of 99% purity costs approximately \$126 500.

Exercise 1A.

1. $\boxed{v = u + at}$

   (a) Find v given that $u = 3$, $a = 5$ and $t = 4$.
   (b) Find v given that $u = 32$, $a = -2$ and $t = 5$.
   (c) Find v given that $u = 2$, $a = 20$ and $t = 2{\cdot}5$.

2. $\boxed{C = 2\pi r}$

   (a) Find C given that $r = 3$.
   (b) Find C given that $r = 15$.
   (c) Find C given that $r = 2{\cdot}8$.

3. $\boxed{V = \frac{4}{3}\pi r^3}$

   (a) Find V given that $r = 3$.
   (b) Find V given that $r = 6$.
   (c) Find V given that $r = 10$.

4. $\boxed{s = \frac{(u + v)}{2}t}$

   (a) Find s given that $u = 3$, $v = 5$ and $t = 10$.
   (b) Find s given that $u = 12$, $v = -3$ and $t = 8$.
   (c) Find s given that $u = 22$, $v = 16$ and $t = 7$.

5. $\boxed{A = 400\,(1{\cdot}12)^n}$

   (a) Find A given that $n = 5$.
   (b) Find A given that $n = 8$.
   (c) Find A given that $n = 10$.

6. A particular industrial process involves the mining and production of a particular metal to various levels of purity. The production of 1 kg of the metal of p% purity costs the company \$$C$ where a good approximation of C is given by the formula:

$$C = \frac{1000}{100 - p}$$

Find (a) the value of C when $p = 90$,
(b) the cost of producing one kg of the metal of 99% purity,
(c) the cost of producing one kg of the metal of 99·9% purity.

7. A particular industrial process involves the mining and production of a particular metal to various levels of purity. The production of 1 tonne of the metal of p% purity costs the company \$$C$ where a good approximation of C is given by the formula:

$$C = 7800 + \frac{80000}{100 - p}$$

Find (a) the value of C when $p = 60$,
(b) the cost of producing one tonne of the metal of 75% purity,
(c) the cost of producing one tonne of the metal of 95% purity.

8. As divers dive to greater depths the pressure on the divers increases due to the weight of the increasing amount of water above them. The pressure, P, in Newtons per square metre, N/m^2, is given by the formula

$P = 9{\cdot}8xd$ where x metres is how far the diver is below the surface of the water,
and d kg/m^3 is the density of the water.

Find, in N/m^2,
(a) the pressure at a depth of 5 metres in liquid of density 1000 kg/m^3,
(b) the pressure at a depth of 10 metres in liquid of density 1030 kg/m^3,
(c) the pressure at a depth of 30 metres in liquid of density 1030 kg/m^3.

9. A scientist suggests that for a particular drug the dose (d units) that should be given to a child, aged c years, can be calculated from the dose (D units) that would be given to an adult, using the rule: $d = \frac{(c+1)}{19} D$.

(a) If the adult dose is 15 units what would be the dose, to the nearest half unit, for a child aged (i) 5 years,
(ii) 10 years,
(iii) 15 years.

(b) The rule makes sense for c up to a certain value. What value is that and why does the rule make no sense after that?

10. To find the density of a solid or liquid we divide its mass by its volume.

i.e. $\text{Density } (d) = \dfrac{\text{Mass } (m)}{\text{Volume } (V)}$.

If the mass is in kilograms and the volume in cubic metres the density will be in kg/m^3. If the mass is in grams and the volume in cubic centimetres the density will be in g/cm^3, etc.

(a) Find the density of Aluminium if 100 cm^3 has a mass of 270 g.

(b) Find the density of Lead if 0·2 m^3 has a mass of 2270 kg.

(c) Find the density of Diamond if 0·2 cm^3 has a mass of 0·702 g.

The density of sea water increases with depth due to the pressure of the water above. Find the density of sea water in g/cm^3

(d) just below the surface where 5 cm^3 has a mass of 5·141 g,

(e) at a depth of 1 000 m where 5 cm^3 has a mass of 5·164 g,

(f) at a depth of 10 000 m where 5 cm^3 has a mass of 5·355 g.

11. If you deposit $\$P$ in a bank account that earns interest at x % per annum, compounded annually, the amount in the account after t years will be $\$A$ where

$$A = P\left(1 + \frac{x}{100}\right)^t .$$

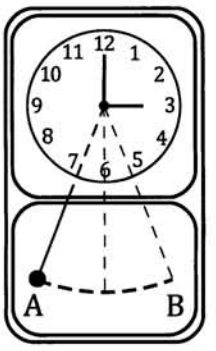
Find the amount in the account after 5 years if:

(a) \$300 is deposited at 10% per annum compounded annually,

(b) \$500 is deposited at 12% per annum compounded annually,

(c) \$12 000 is deposited at 7% per annum compounded annually.

12. The pendulum of the clock shown on the right performs one cycle as it swings from A to B and back to A again. The time taken for a pendulum of length ℓ metres to perform one cycle is T seconds where $T = 2\pi\sqrt{\dfrac{\ell}{9{\cdot}8}}$.



Find the value of T (correct to two decimal places) for a pendulum of length (a) 1·15 m,

(b) 40 cm,

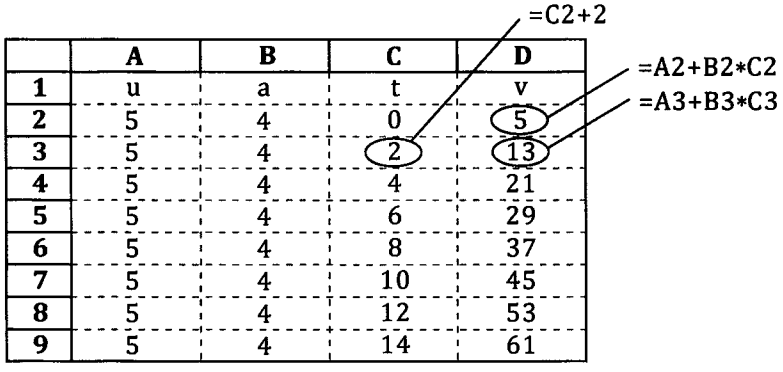
(c) 20 cm.

Use of formulae in spreadsheets.

In a spreadsheet entries are made into a table arrangement involving rows and columns. The entries can then be manipulated and used in calculations.
The spreadsheet shown below has values for u, a and t in columns A, B and C respectively, as indicated by the use of row 1 for appropriate labels. The entries in column D are calculated from the values for u, a and t according to the formula v = u + at. For cell D2 the formula entered in the cell is:

=A2+B2*C2

Note
- Spreadsheet formulae always start with =.
- Appropriate formulae for cells D3 to D9 can be entered using the formula in D2 together with the ability of spreadsheets to "fill down".
  This will place the formula =A3+B3*C3 in cell D3,
  =A4+B4*C4 in cell D4, etc.
- Spreadsheets recognise the symbol * as representing multiplication.



	A	B	C	D
1	u	a	t	v
2	5	4	0	5
3	5	4	2	13
4	5	4	4	21
5	5	4	6	29
6	5	4	8	37
7	5	4	10	45
8	5	4	12	53
9	5	4	14	61

Create such a spreadsheet yourself for the values of u, a and t shown above.

Spreadsheets can be especially useful for accounting purposes. The spreadsheet below shows the various deposits and withdrawals from a bank account over a period of time. Column E, when complete, will show the balance after each transaction.

	A	B	C	D	E
1	Date	Details	Withdrawals ($)	Deposits ($)	Balance ($)
2	1-Mar	Opening balance			1245.65
3	2-Mar	Pay cheque		1243.65	2489.3
4	3-Mar	Rent	480		
5	5-Mar	Shoes	80.9		
6	7-Mar	Supermarket	92.45		
7	8-Mar	Cash out	200		
8	10-Mar	Cheque refund		17.5	
9	10-Mar	Credit card payment	245		
10	14-Mar	Supermarket	185.6		
11	15-Mar	Cash out	200		
12	16-Mar	Pay cheque		1243.65	

Create the above spreadsheet yourself and complete it.

The spreadsheet below shows the marks obtained by 12 students in the ten assessment items of a college course.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Name	Item	Item	Item	Item	Item	Item	Item	Item	Item	Item	Total
2		1	2	3	4	5	6	7	8	9	10	out of 100
3	Sally A	6	7	8	18	17	24	16	11	32	7	
4	Chris C	4	6	6	14	11	14	9	7	26	8	
5	Charlie C	7	10	9	21	19	18	16	17	32	10	
6	Ying H	9	9	7	24	17	23	14	14	37	11	
7	Su J	5	6	6	13	18	17	13	12	23	7	
8	Sanji L	8	9	6	13	16	19	16	17	22	9	
9	Connie N	8	5	6	13	13	14	13	11	22	8	
10	Michele R	7	7	7	18	13	19	10	11	29	9	
11	Becky S	3	5	5	10	12	10	12	9	23	5	
12	Chris T	9	7	10	24	19	22	17	18	33	14	
13	Duane W	10	10	8	20	20	23	17	14	31	15	
14	Icolyn Y	9	8	8	17	14	19	15	16	24	11	
15												
16		10	10	10	25	25	25	20	20	40	16	100

Column L is to show the total mark for each student out of 100, with each of the ten assessment items counting for 10 marks in this total of 100.

Write a suitable formula for cell L3.
Create the above spreadsheet yourself and complete it.

The spreadsheet below is for three items, 1, 2 and 3, that a company sells for $25 each, $30 each and $35 each respectively. The spreadsheet shows orders numbered 6001 (for 3 item 1s, 2 item 2s and 1 item 3), 6002 (for 2 item 1s and 2 item 3s), etc. Notice that column E gives the total cost of the order, column F gives the 10% Goods and Services Tax (GST), column G gives the cost + GST total and columns H, I and J give the remaining stock numbers of each item, starting from the initial stock level of 500 for each item.

	A	B	C	D	E	F	G	H	I	J
1	Order#	Item 1	Item2	Item3	Cost	GST	Total	500	500	500
2	6001	3	2	1	$170.00	$17.00	$187.00	497	498	499
3	6002	2	0	2	$120.00	$12.00	$132.00	495	498	497
4	6003	0	1	5	$205.00	$20.50	$225.50	495	497	492
5	6004	2	2	2	$180.00	$18.00	$198.00	493	495	490

Create such a spreadsheet yourself for orders 6001 to 6010 where 6005 to 6010 are as given below.

Order	Item 1	Item 2	Item 3	Order	Item 1	Item 2	Item 3
6005	5	4	5	6006	3	7	4
6007	2	1	6	6008	0	20	11
6009	1	3	3	6010	7	0	9

Miscellaneous Exercise One.

This exercise may include questions involving the work of this chapter and the ideas mentioned in the preliminary section at the beginning of the book.

1. Evaluate each of the following:

(a)	$(-4)^2$	(b)	$2 + (-3) \times (-4)$	(c)	$2^3 + 3^2$
(d)	$(2^3 + 2)^2$	(e)	$6 - 5 \times (-4)$	(f)	$(-1)^5$
(g)	$(-1)^6$	(h)	$\frac{8+4}{4}$	(i)	$\frac{14 \cdot 14}{10}$
(j)	$\frac{16 \cdot 16}{8}$	(k)	$1 + (15 - 3 \times 4)^2$	(l)	$6 - -4 \div 2$

2. Find the value of each of the following expressions given that $x = 5$.

(a)	$x + 2$	(b)	$2x$	(c)	x^2
(d)	$2x + 3$	(e)	$3x + 2$	(f)	x^3
(g)	$4x - 1$	(h)	$1 - 4x$	(i)	$2(3 + x)$
(j)	$(3 + x)^2$	(k)	$(x - 3)^3$	(l)	$4(2x - 3)$
(m)	$\frac{x+3}{2}$	(n)	$\frac{4x-2}{3}$	(o)	$\frac{15}{x-2}$
(p)	$\sqrt{3x+1}$	(q)	$\sqrt{8x-4}$	(r)	$\sqrt[3]{2x-2}$

3. Without using your calculator, use appropriate rounding to determine estimates for each of the following, showing your method.

(a) 208×84 (b) $19{\cdot}6 \times 4{\cdot}7$ (c) $\frac{208}{9{\cdot}7}$ (d) $\frac{4864}{103}$

(e) Attempting to estimate how far it was from one location to another a person counted the number of paces they took in walking from one to the other. They found it required 623 paces. The person measured a typical pace as being approximately 80 cm. Estimate the distance between the two locations.

4. One way of categorising a person as overweight, correct weight or underweight is to consider their "mass on height squared index", I, also called the body mass index or BMI.

This index is given by $I = \frac{m}{h^2}$ where m kg is the mass of the person and h metres is their height.

If $I < 20$ the person is considered underweight,
If $20 \leq I \leq 25$ the person is considered to be the correct weight,
If $I > 25$ the person is considered overweight.

Classify each of the following people

(a) Julie, 170 cm, 65 kg, (b) Alex, 1·95 m, 60 kg,
(c) Bill, 195 cm, 97 kg, (d) Betty, 1·65 m, 55 kg.

Construct a spreadsheet that will automatically calculate the body mass index of a person given their mass and height. Include the four people mentioned above in your spreadsheet.

Challenge: Can you get your spreadsheet to allocate the classifications too?