Chapter 9.

Surface area and volume.

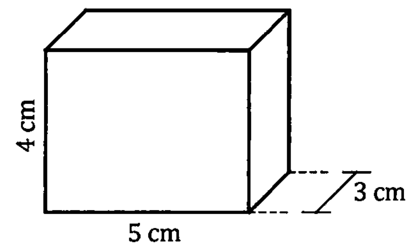
Surface area.

The *Preliminary work* section at the beginning of this book included a reminder of how to determine the area of rectangles, triangles, parallelograms, trapeziums and circles. Chapter eight then required you to use these skills in various situations.

We can use this ability to determine the area of various shapes to determine the total surface area of prisms, cylinders and pyramids.

For example, for the solid rectangular prism shown sketched below.

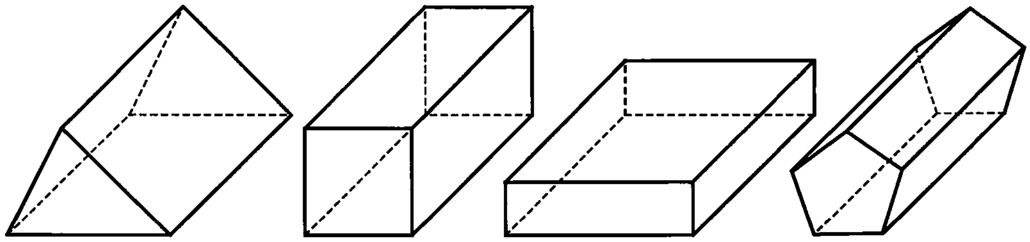
Face	Area
Front	20 cm^2
Back	20 cm^2
Top	15 cm^2
Base	15 cm^2
Left side	12 cm^2
Right side	12 cm^2
Total surface area:	94 cm^2



The prism has a total surface area of 94 cm^2.

Prisms.

- A prism is a three dimensional shape with all of its faces polygons and with the same shape cross section all along its length.
- Prisms are named according to their uniform cross section. However we do not tend to call a cube a square prism because "cube" tells us that <u>all</u> the faces are square. Not all square prisms are cubes.

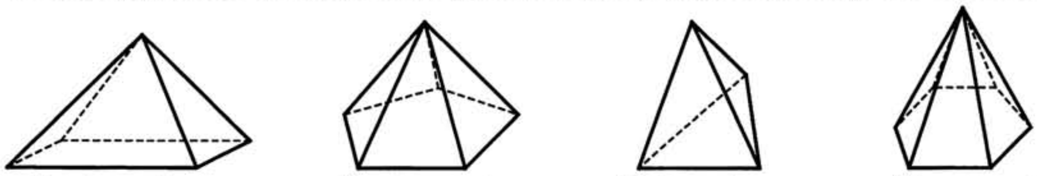


Triangular prism | Square prism | Rectangular prism | Pentagonal prism

- If all of the sides of a prism go back perpendicularly from the front face all of the sides will be rectangular. Such prisms are said to be *right* prisms. Assume all of the prisms in this unit are right prisms.
- If a three dimensional shape has a circular uniform cross section it is a **cylinder**. Whilst a cylinder is sometimes thought of as a circular prism, technically it is not a prism because its faces are not polygons.

Pyramids.

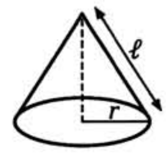
- A three dimensional shape that has a polygon as its base and all the sides meeting at a point is called a pyramid. We name the pyramid according to the shape of its base:



Rectangular pyramid | Pentagonal pyramid | Triangular pyramid | Hexagonal pyramid

- As with prisms we can determine the total surface area of a pyramid by summing the areas of the separate surfaces.
- If a three dimensional shape has a circular base and then comes up to a point it is a **cone**. Whilst a cone is sometimes thought of as a circular based pyramid, technically it is not a pyramid because the base is not a polygon.

For a solid cone with base radius r and slant height ℓ, see diagram, the curved surface area and total surface area are given by the rules:



$$\text{Curved surface area} = \pi r \ell$$

$$\text{Total surface area} = \pi r \ell + \pi r^2$$

Spheres.

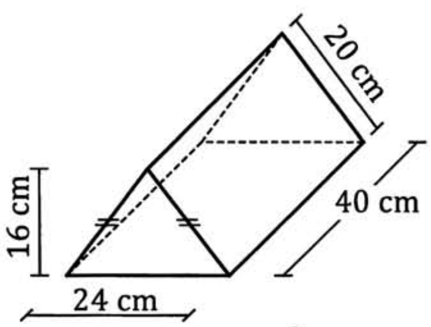
One common three dimensional shape not covered so far in this chapter is the sphere. The surface area of a sphere of radius r is given by:

$$\text{Surface area} = 4\pi r^2$$

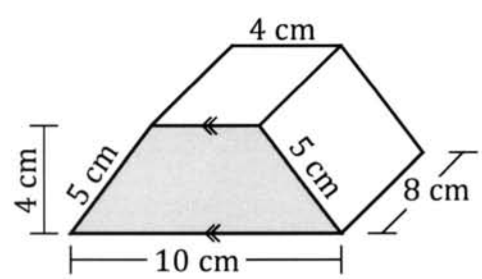
Example 1

Find the surface area of each of the following solid bodies.

(a) (Triangular prism)



(b) (Trapezoidal prism)



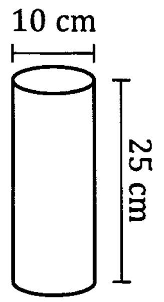
(a) Base: 960 cm^2
Ends: $2 \times 192\text{ cm}^2$
Sides: $2 \times 800\text{ cm}^2$
Total: $2\,944\text{ cm}^2$
The triangular prism has a total surface area of $2\,944\text{ cm}^2$.

(b) Base: 80 cm^2
Top 32 cm^2
Ends $2 \times 28\text{ cm}^2$
Sides $2 \times 40\text{ cm}^2$
Total: 248 cm^2
The trapezoidal prism has a total surface area of 248 cm^2.

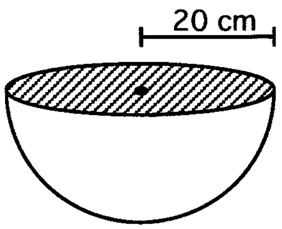
Example 2

Find the surface area of each of the following solid bodies.

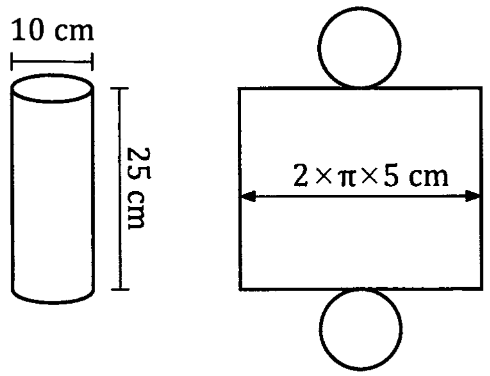
(a) (Cylinder)



(b) (Hemisphere)



(a) "Unrolling" the cylinder we see that its surface area consists of three parts: Two circles and a rectangle.



Area of the two circles: $2 \times \pi \times 5^2$ cm^2

Area of the rectangle: $2 \times \pi \times 5 \times 25$ cm^2

Total: 942·5 cm^2 (1 d.p.)

The triangular prism has a total surface area of 942·5 cm^2 correct to 1 decimal place.

(b) Surface area of sphere = $4\pi r^2$

Thus for the hemisphere:

Area of curved surface: $2 \times \pi \times 20^2$ cm^2

Area of flat surface: $\pi \times 20^2$ cm^2

Total: 3 770 cm^2 (nearest cm^2)

The hemisphere has a total surface area of 3 770 cm^2, to the nearest cm^2.

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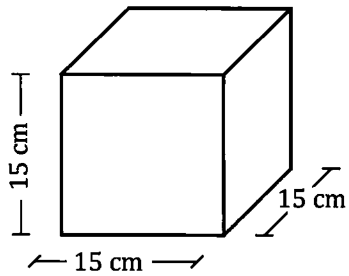
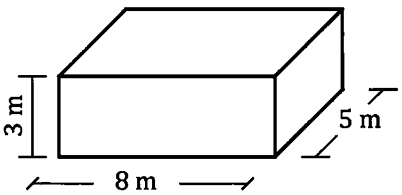
Exercise 9A

Find the surface area of each of the following solids shown in numbers 1 to 16.

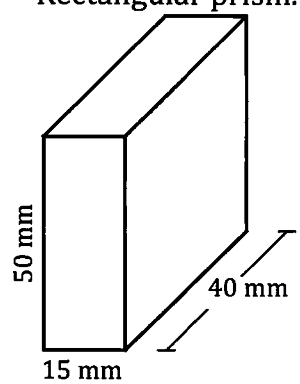
1. Cube.



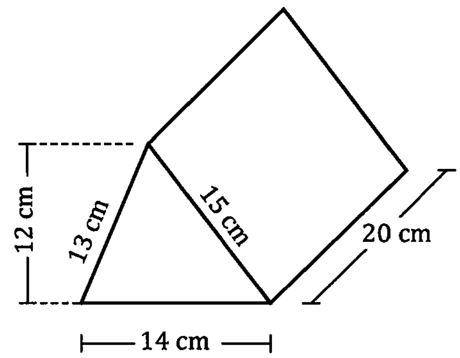
2. Rectangular prism.



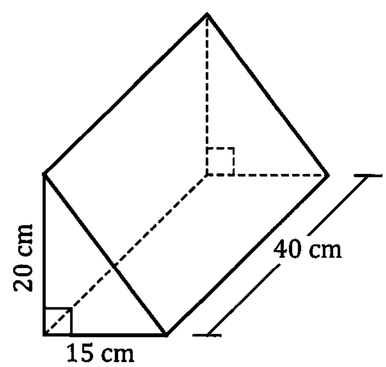
3. Rectangular prism.



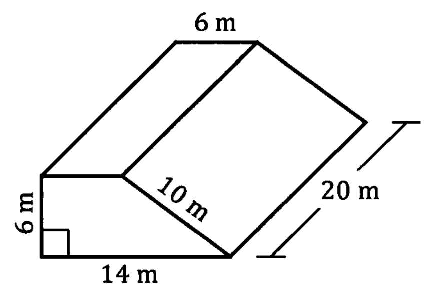
4. Triangular prism.



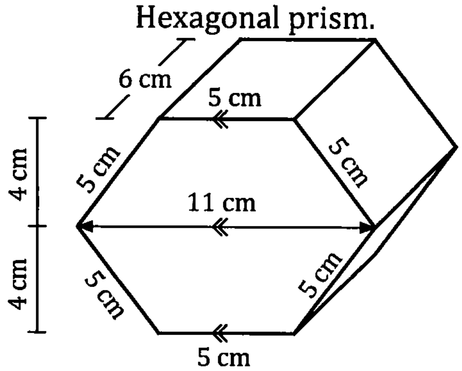
5. Triangular prism.



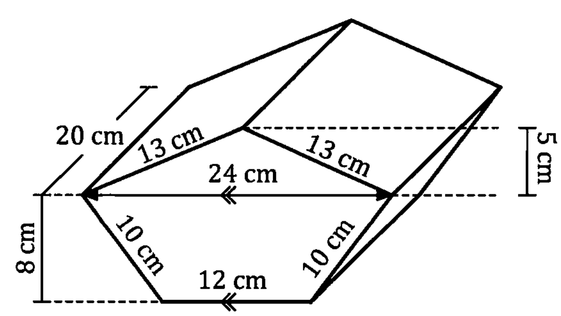
6. Trapezoidal prism.



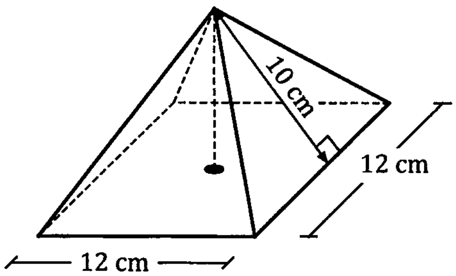
7. Hexagonal prism.



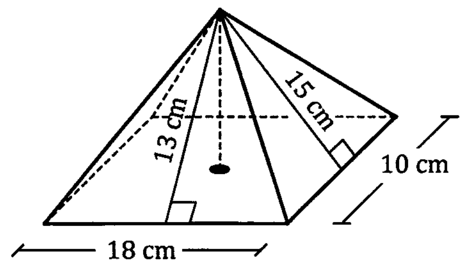
8. Pentagonal prism.



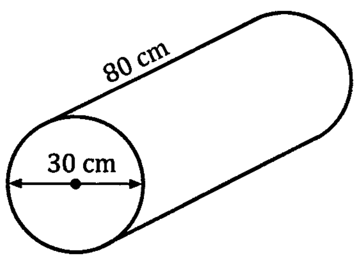
9. Square pyramid with top point vertically above centre of base.



10. Rectangular pyramid with top vertically above centre of base.

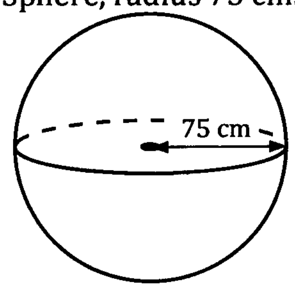


11. Cylinder.



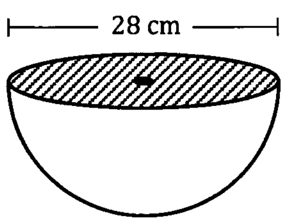
Give answer to nearest 1 cm^2.

12. Sphere, radius 75 cm.



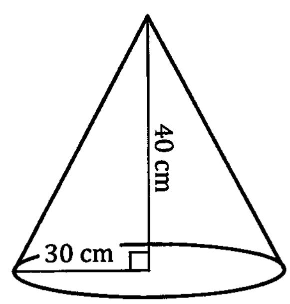
Give answer to nearest 100 cm^2.

13. Hemispere.



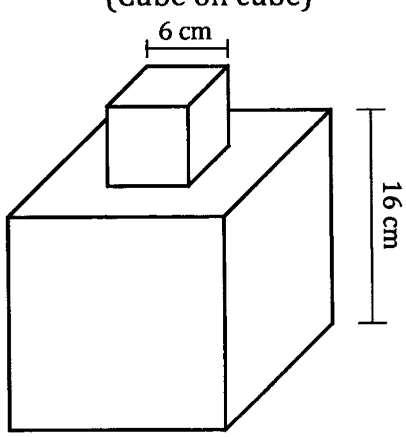
Give answer to nearest 50 cm^2.

14. Cone.

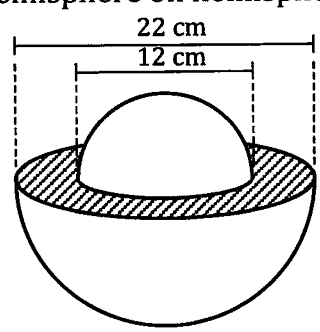


Give answer to nearest 100 cm^2.

15. (Cube on cube)



16. (Hemisphere on hemisphere)



Give answer to nearest cm^2.

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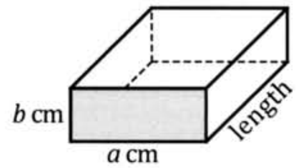
Volume.

For a rectangular prism as shown on the right:

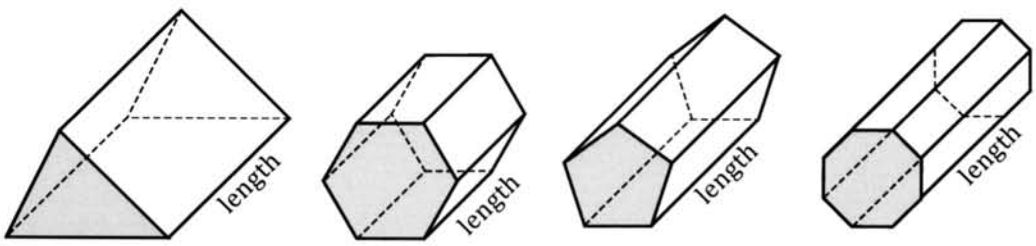


$$\begin{aligned}\text{Volume} &= a \text{ cm} \times b \text{ cm} \times \text{length}\\ &= ab \text{ cm}^2 \times \text{length}\\ &= \text{area of front face} \times \text{length}\end{aligned}$$

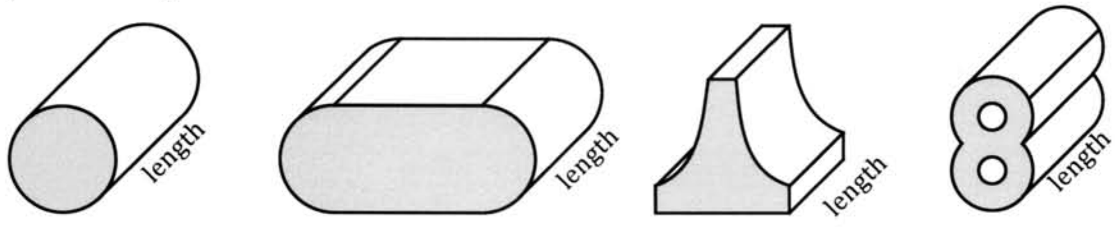
The uniform cross section of triangular prisms, pentagonal prisms and many other prisms are <u>not</u> rectangular. However we can find the volume of such prisms in the same way as we can for rectangular prisms, i.e. find the area of the uniform cross section and then multiply by the length of the prism.

Thus for all of the right prisms shown below:

Volume of prism = Area of uniform cross section × length



This rule can also be used to determine the volume of other shapes which, because they have some faces that are <u>not</u> polygons, are not prisms. The rule can be used provided the shape has a uniform cross section and the "body" of the shape goes back perpendicularly from the uniform cross section.



Thus for all of the above shapes:

Volume = Area of uniform cross section × length

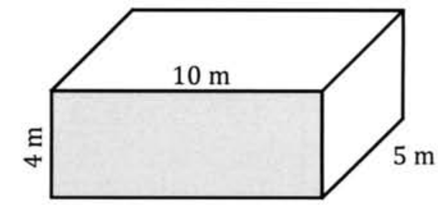
To determine the volume of a pyramid we use the rule:

$$\textbf{Volume of pyramid (or cone)} = \frac{\textbf{base area} \times \textbf{height}}{3}$$

For a sphere the rule is:

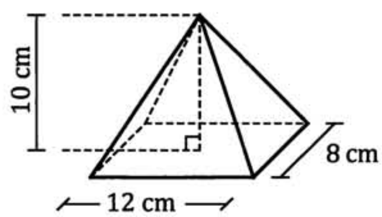
$$\textbf{Volume of sphere of radius r} = \frac{4}{3}\pi r^3$$

Example 3
Find the volume of the rectangular prism shown on the right.



$$\begin{aligned}\text{Volume} &= \text{shaded area} \times \text{length}\\ &= 4\text{ m} \times 10\text{ m} \times \text{length}\\ &= 40\text{ m}^2 \times 5\text{ m}\\ &= 200\text{ m}^3\end{aligned}$$

Example 4
Find the volume of the rectangular based pyramid shown on the right.



$$\begin{aligned}\text{Volume} &= \frac{\text{base area} \times \text{height}}{3}\\ &= \frac{96\text{ cm}^2 \times 10\text{ cm}}{3}\\ &= 320\text{ cm}^3\end{aligned}$$

The pyramid has a volume of 320 cm^3.

$96 \times 10 \div 3$

320

Example 5
Find the volume of a sphere of radius 2·41 m giving your answer to the nearest cubic metre.

$$\begin{aligned}\text{Volume of sphere} &= \frac{4}{3}\pi r^3\\ \text{Hence volume} &= \frac{4}{3} \times \pi \times 2{\cdot}41^3\text{ m}^3\\ &= 58{\cdot}6\text{ m}^3 \text{ (to 1 d.p.)}\end{aligned}$$

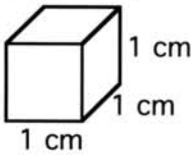
The sphere has a volume of 59 m^3 (to the nearest cubic metre).

$\frac{4}{3} \times \pi \times 2.41^3$

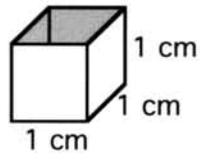
58.63267886

Volume and capacity.

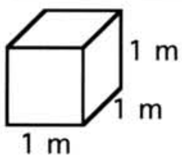
Whilst **volume** is the amount of space a solid occupies, **capacity** is the amount a container can hold. We measure the volume of a solid in mm^3, cm^3 and m^3 and we tend to measure the capacity of containers and the volume of a liquid in millilitres, litres and kilolitres.



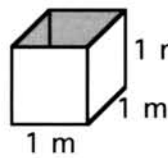
The block shown above has a volume of
1 cubic centimetre (1 cm^3)



The container shown above has a **capacity** of
1 millilitre (1 mL)



The block shown above has a volume of
1 cubic metre (1 m^3)



The container shown above has a **capacity** of
1 kilolitre (1 kL)

1 L = 1000 mL
1 L of liquid occupies 1 000 cm^3

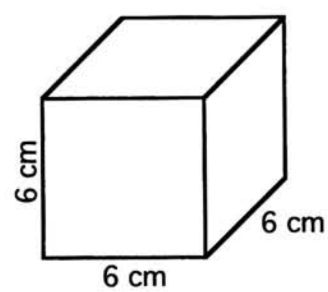
1 kL = 1000 L
1 kL of liquid occupies 1 m^3

Whilst the units for capacity are based on the litre it is sometimes the case in real life that capacity is given using cm^3 or m^3. For example the capacity of a motor bike engine may be quoted in "ccs", standing for cubic centimetres. The capacity of a waste removal bin may be quoted in m^3.

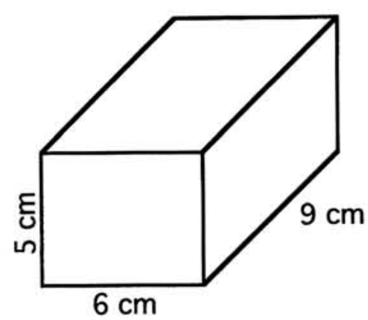
Exercise 9B

Find the volume of each of the following solids.

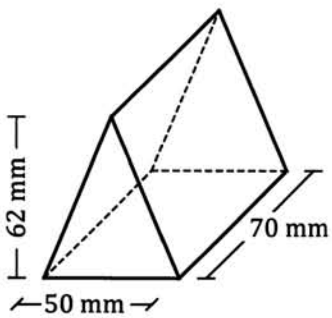
1. Cube.



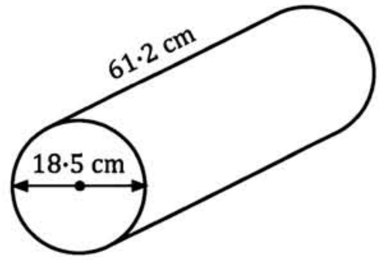
2. Rectangular prism.



3. Triangular prism.

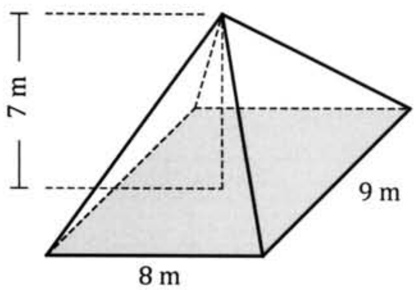


4. Cylinder.

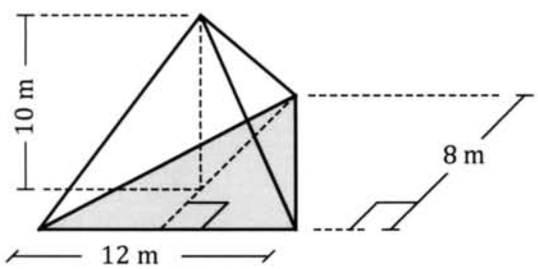


Give your answer to the nearest 50 cubic centimetres.

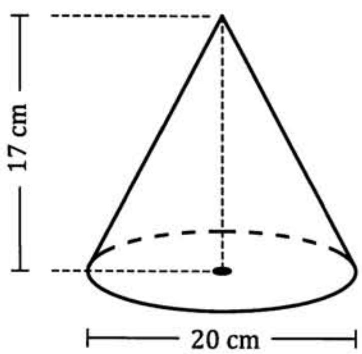
5. Rectangular pyramid.



6. Triangular pyramid.

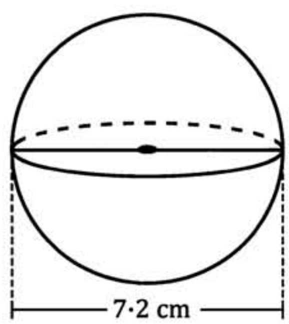


7. Cone.



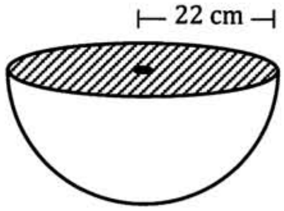
Give your answer to the nearest cubic centimetre.

8. Sphere.



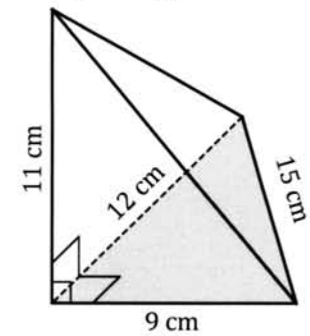
Give your answer to the nearest cubic centimetre.

9. Hemisphere.



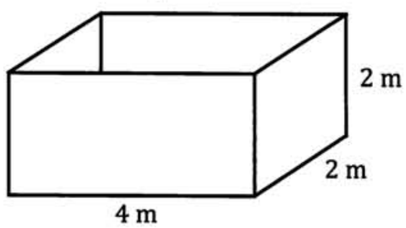
Give your answer to the nearest 100 cubic centimetres.

10. Triangular pyramid.

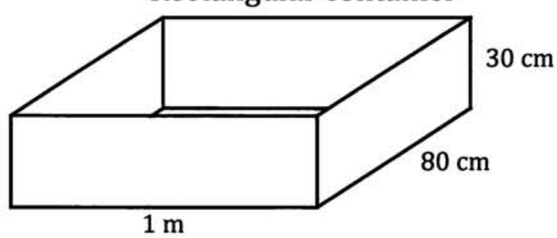


Find the capacity of each of the following containers:

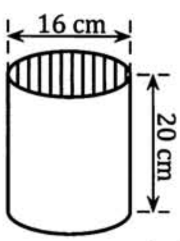
11. Rectangular container



12. Rectangular container

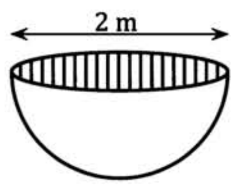


13. Cylindrical container



Give your answer to the nearest millilitre.

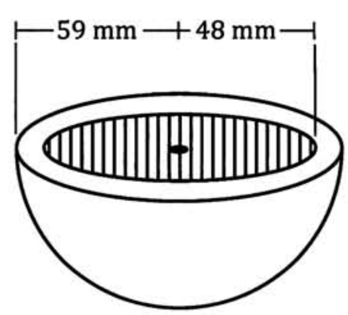
14. Hemispherical container



Give your answer to the nearest litre.

15. A solid metal cube with all edges of length 20 cm is to be melted down and recast into smaller cubes each with an edge length of 1 cm. How many such cubes could be made?
If instead the original cube were to be recast into spheres of radius 1 cm how many such spheres could be made?

16. Find the volume of material required to make the hemispherical shell shown on the right with the radius of the internal hemispherical "space" and of the external hemisphere as shown.
Give your answer rounded **up** to the next 100 mm^3.



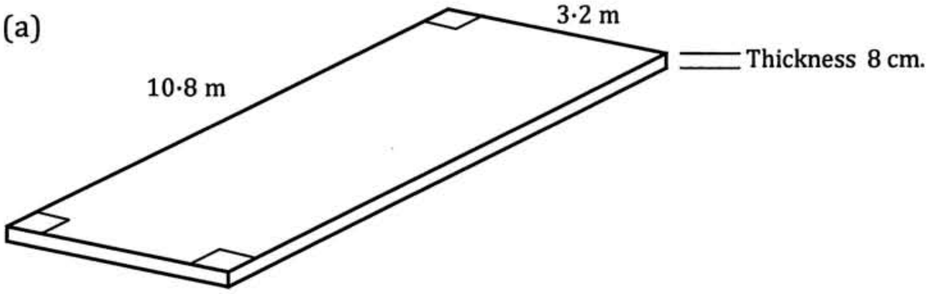
Applications.

In the following exercise you will again need to determine various surface areas, volumes and capacities. However the questions now involve some everyday contexts for which determining these quantities is significant.

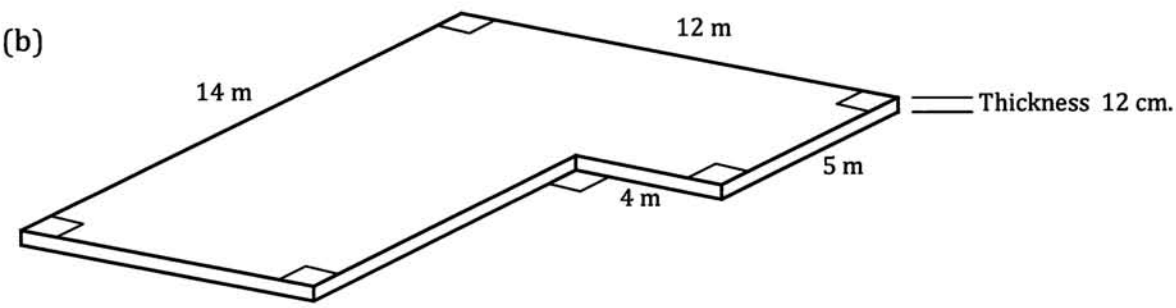
Exercise 9C.

1. Determine the volume of concrete required to make each of the following concrete slabs. Give each answer in cubic metres rounded to one decimal place.

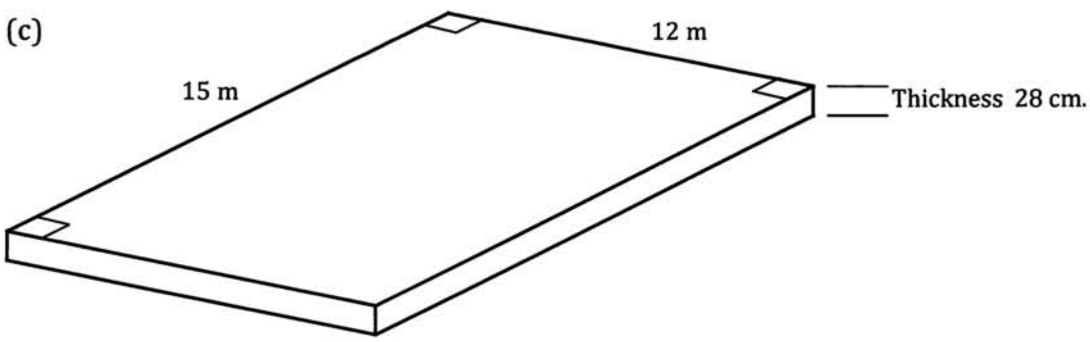
(a)



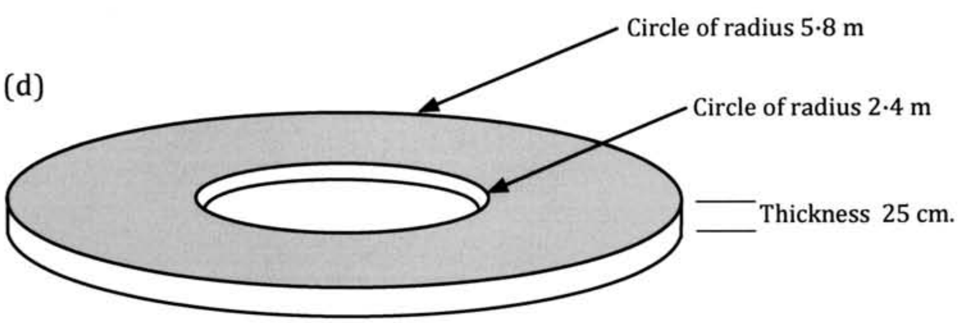
(b)



(c)

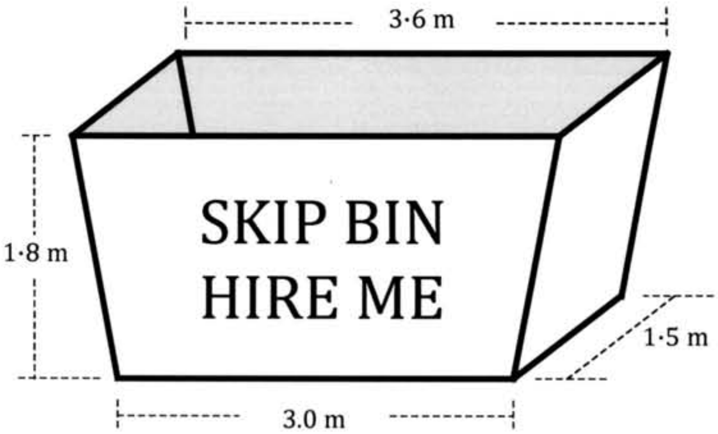


(d)



 ISBN 9780170350440.

2. A skip hire company has skips of various sizes available for hire.
One of the available sizes is as shown on the right.

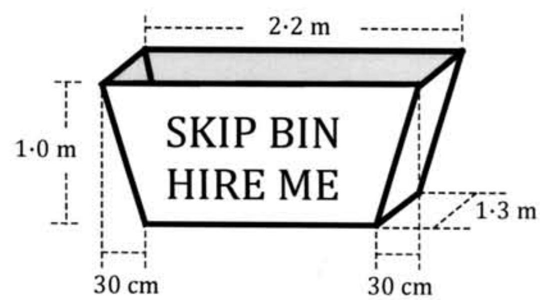


Confirm that with the dimensions shown the volume of this skip, when filled level to the rectangular top, is $8{\cdot}91\ \text{m}^3$.

However, because the thickness of the walls makes each dimension of the fillable space somewhat less than the external dimensions given, the company advertises this skip as being an $8\ \text{m}^3$ skip.

For each of the following skips determine the volume of each, when filled level to the rectangular top, according to the dimensions shown, and then suggest what size the company is likely to advertise the skip as being.

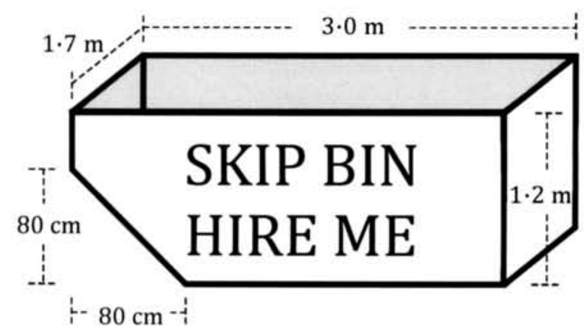
(a)



(b)

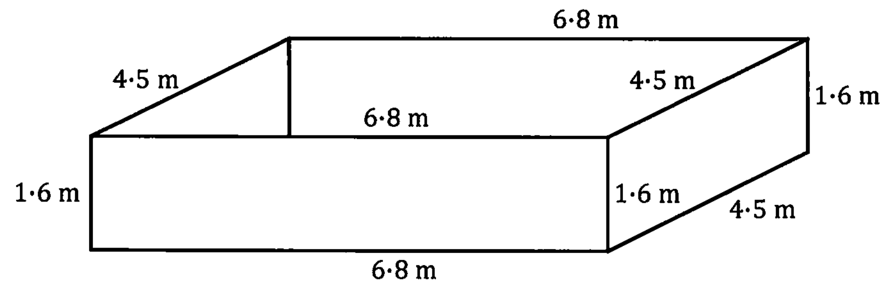
(c)

(d)

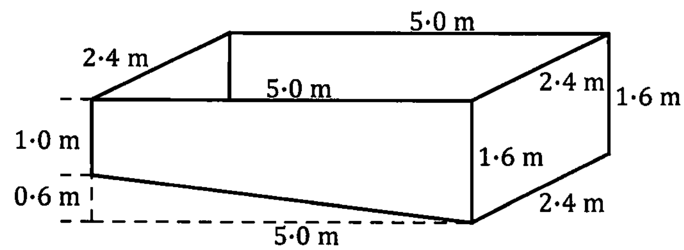


3. Find the volume of water needed to fill each of the following swimming pools to a level that is 10 cm below the top of the pool. (Assume that the measurements given in the diagrams are internal dimensions.)
How long would it take to fill each pool using a water supply that delivers 50 litres per minute?

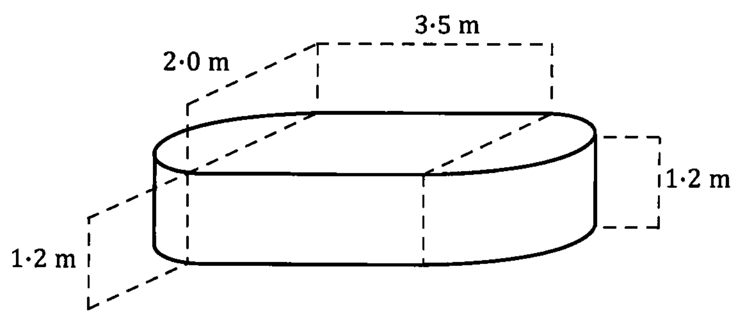
(a)



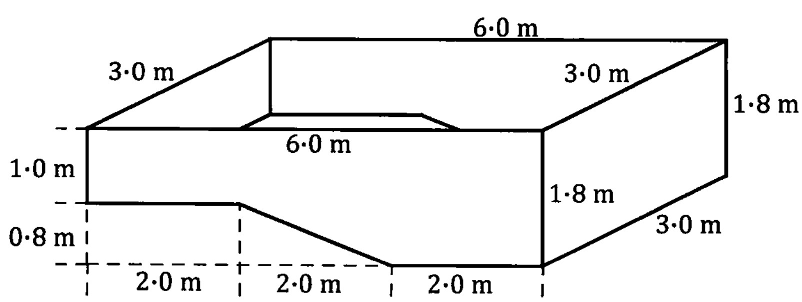
(b)



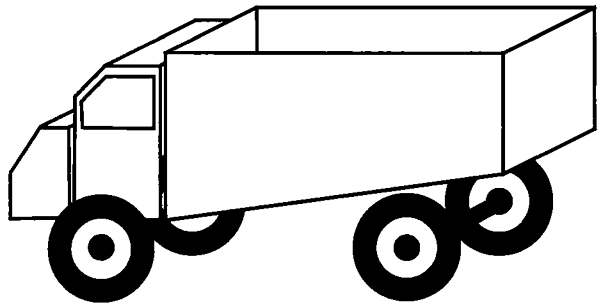
(c) The floor of this pool is a rectangle with a semi-circle at each end.
The pool is of uniform depth.



(d)



4. A company makes containers of various sizes that can fit onto trucks which are then used for transporting loads of soil or sand or similar material, for example at mine sites, building sites etc.
Neglecting the thickness of the steel used to make the container determine the volume of material each of the following containers can hold for both

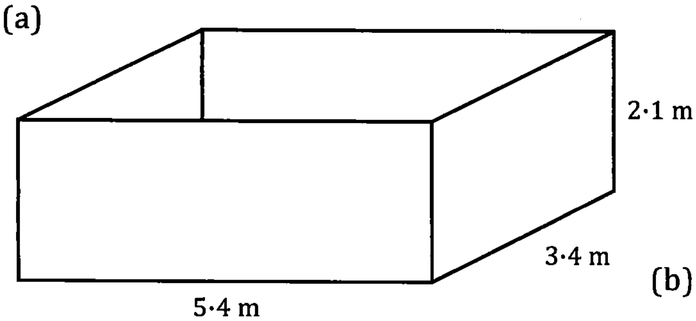


(i) a load that is level with the top of the container,

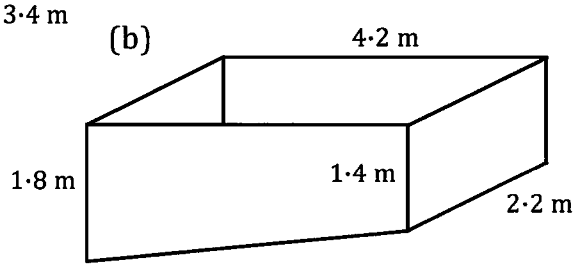
(ii) a heaped load – which increases the amount that can be carried by 25% for container (a), 30% for containers (b) and (c) and 35% for container (d)

giving all answers in cubic metres rounded to one decimal place.

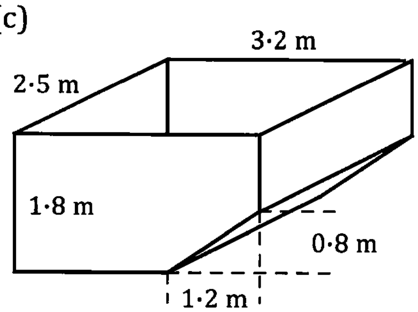
(a)



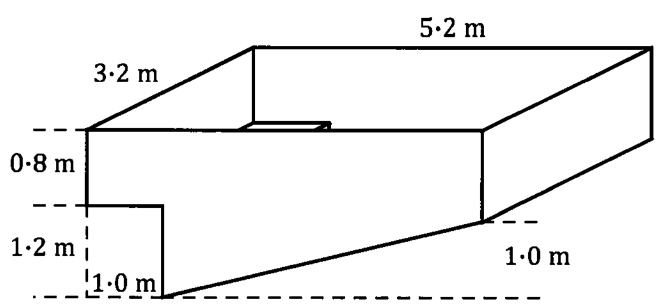
(b)



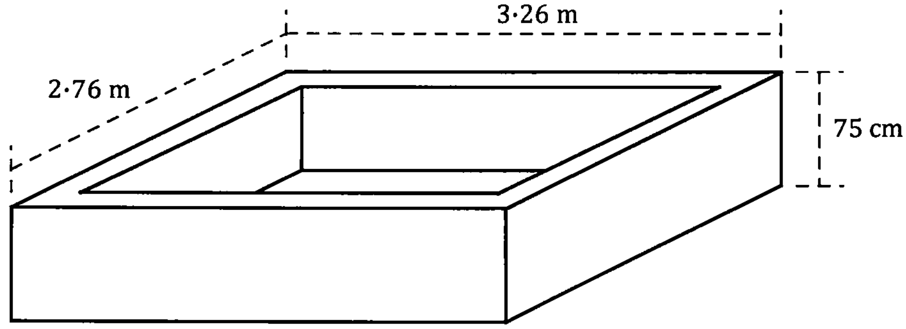
(c)



(d)



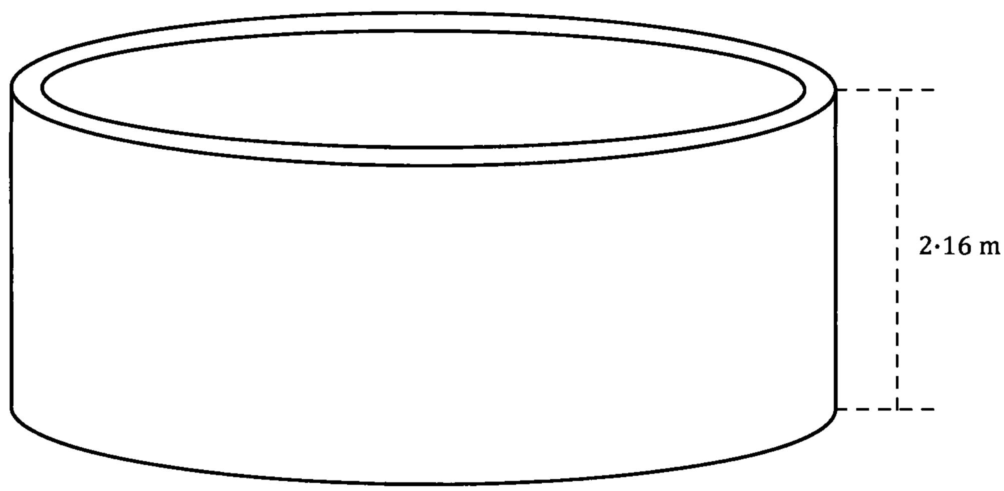
5. (a) The water tank shown below is to be constructed in concrete, with concrete walls and a concrete base. A metal lid is to be put in place later.



- ☞ The concrete base is to be 150 mm thick
- ☞ The walls are to be 130 mm thick.

Determine the capacity of the water tank and the volume of concrete required to make it.

(b) The cylindrical water tank shown below is to be constructed in concrete, with the circular base of diameter 5·4 metres. A plastic lid is to be put in place later.

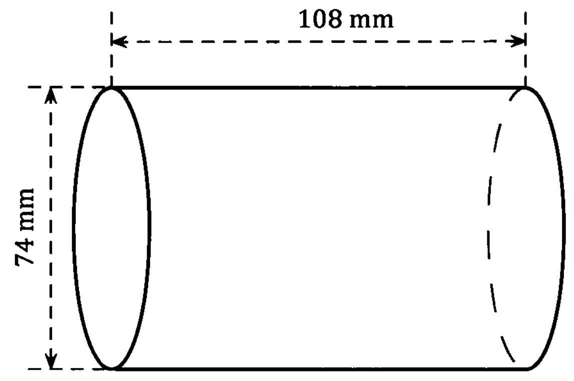


- ☞ The concrete base is to be 200 mm thick
- ☞ The walls are to be 150 mm thick.

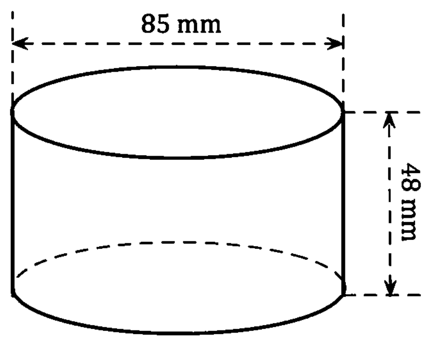
Determine the capacity of the water tank and the volume of concrete required to make it.

ISBN 9780170350440.

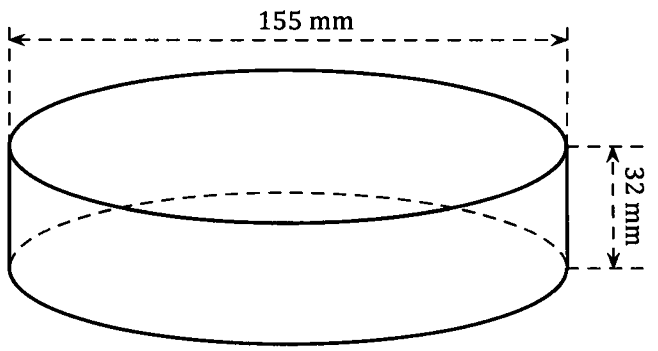
6. To calculate the area of metal sheeting required to make cylindrical metal cans a company calculates the external surface area of the cylinder involved and then adds 10% for joins and wastage.
Calculate the area of metal sheeting required for each of the following production runs (to the nearest m^2 and including base and lid):



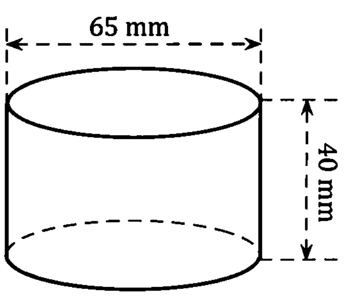
(a) Fifty thousand of the cans shown on the right.



(b) Two thousand of the cans shown on the right.

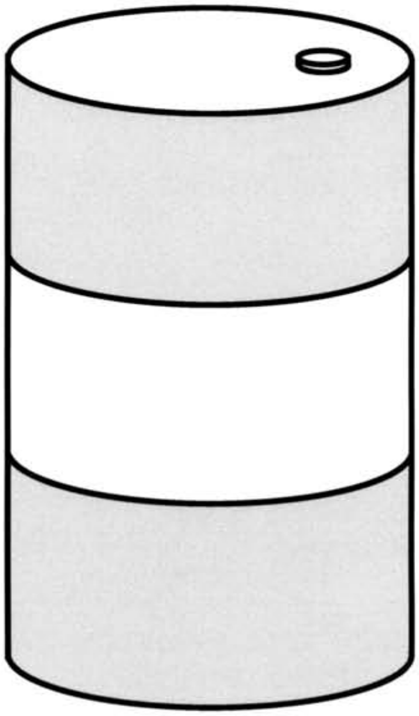


(c) One million of the cans shown on the right.



(d) Five thousand of the cans shown on the right.

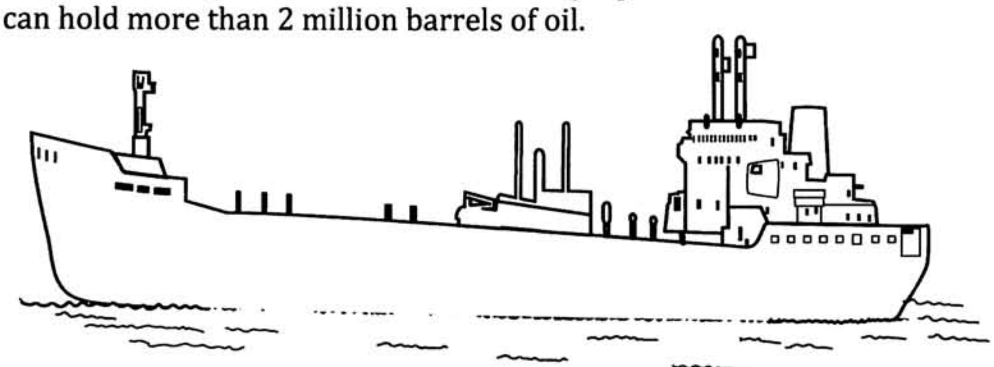
7. (a) Taking the available space inside an oil drum as being cylindrical with a base diameter of 572 millimetres and a height of 851 millimetres determine the capacity of an oil drum in litres, rounded to two decimal places.



(b) In fact 200 litres of oil is placed into each drum. What percentage of the available space is occupied by oil when a drum contains 200 litres of oil (to the nearest 0·5%)?

(c) In the United States of America these oil drums are sometimes referred to as 55-gallon drums but in the United Kingdom they are referred to as 44-gallon drums.
Do some research and write a paragraph or two explaining why this is so.

(d) Some of the big oil tankers that move large quantities of oil around the earth can hold more than 2 million barrels of oil.

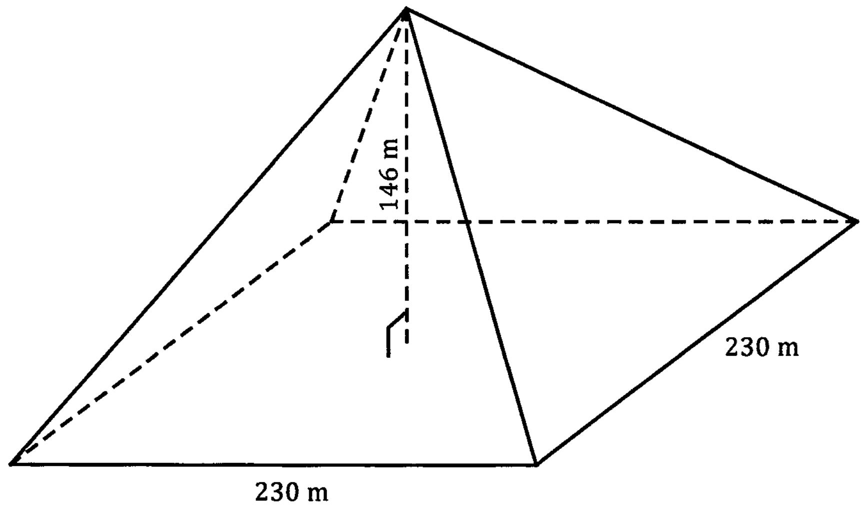


If the oil from 2 million barrels were to form "a cube of oil" what would be the length of each side of the cube?

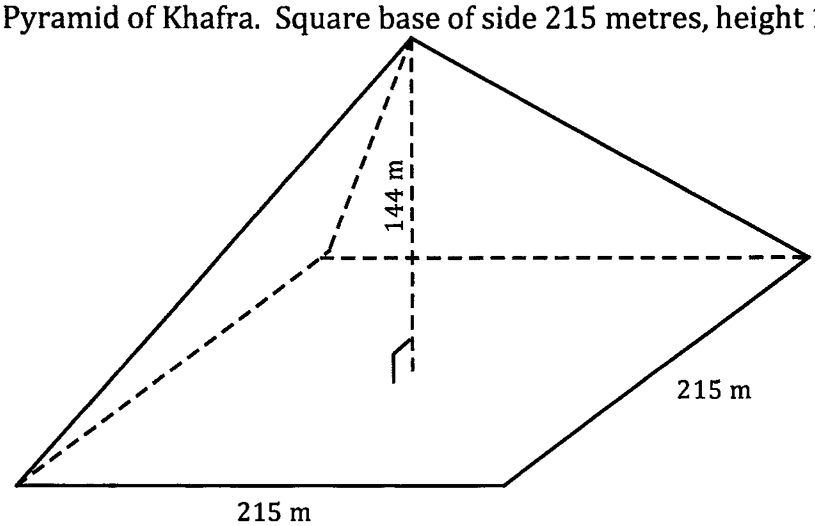
ISBN 9780170350440.

8. For each of the following pyramids find
   (i) the volume of the pyramid,
   (ii) the weight of rock required to make the pyramid assuming that for the rock used each cubic metre has a weight of 2500 kg and that 10% of the volume of each pyramid is empty space (i.e. chambers and corridors).

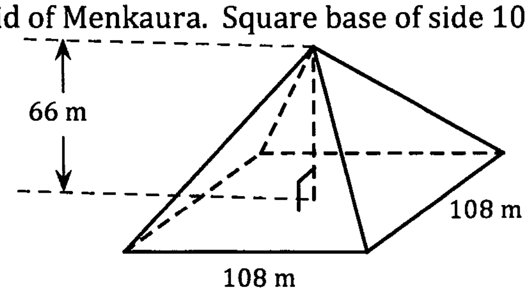
   (a) The Pyramid of Khufu. Square base of side 230 metres, height 146 metres.



   (b) The Pyramid of Khafra. Square base of side 215 metres, height 144 metres.



   (c) The Pyramid of Menkaura. Square base of side 108 m, height 66 m.



9. Find the volume of each of the following spheres.

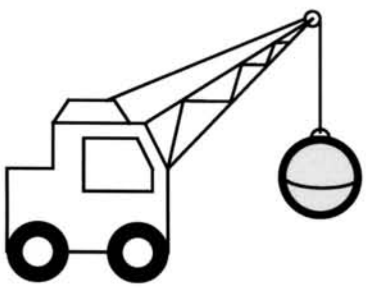
(a)

The earth is approximately a sphere of radius 6400 km.

(b)

A spherical basketball of diameter 24 cm.

(c)

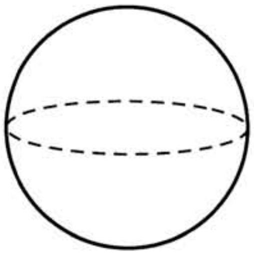


A spherical wrecking ball of radius 45 cm.

(d)

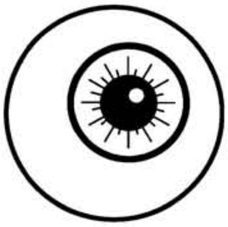
The moon is approximately a sphere of radius 1740 km.

(e)



A snooker ball is a sphere of diameter 52·5 mm.

(f)



An eyeball is approximately a sphere of diameter 2·2 cm.

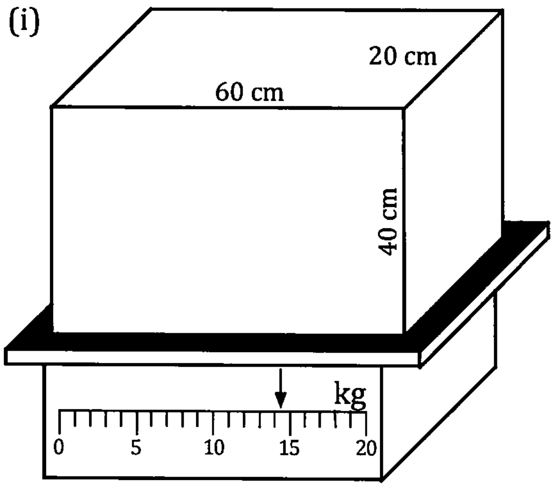
10. Parcel delivering companies usually charge according to the weight of a parcel. However, very large light parcels can take up a lot of room in the delivery vehicle, denying space for other parcels, whilst not bringing in much revenue. To avoid this problem, when asked to deliver a large light parcel, the cost may be based on the parcel's "cubic weight". Each company may have its own formula for calculating their version of the cubic weight. One way involves determining the cubic weight in kilograms by multiplying the parcels volume in cubic metres by 250. By this method, a rectangular parcel with dimensions 80 cm by 50 cm by 40 cm would have a cubic weight of 40 kg (because $0{\cdot}8 \times 0{\cdot}5 \times 0{\cdot}4 \times 250 = 40$). If the real weight of the parcel was less than 40 kg then it would be charged for as if it weighed 40 kg. If it weighed over 40 kg it would be charged for at its real weight.

    Companies may charge for each kg *or part thereof.* A parcel weighing 4·2 kg would be charged as 5 kg. A parcel weighing 17·8 kg would be charged as 18 kg etc.

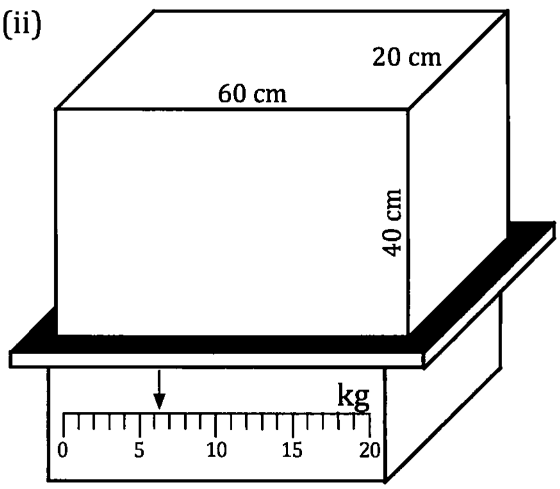
    For each of the following parcels calculate both the real weight and the cubic weight (using the above method), and then calculate the charge.

    (a) Charged at $7·50 for the first kilogram plus $4·40 per kilogram, or part thereof, after that.

    (i)

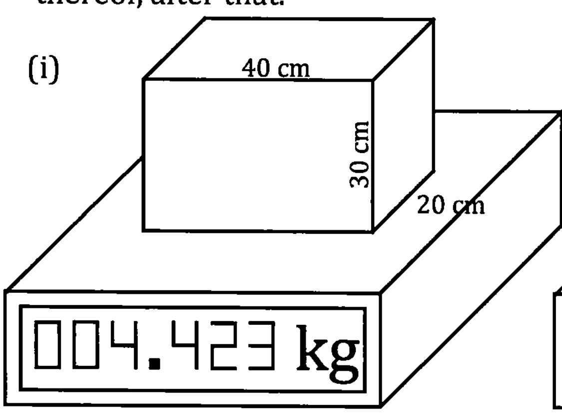


    (ii)

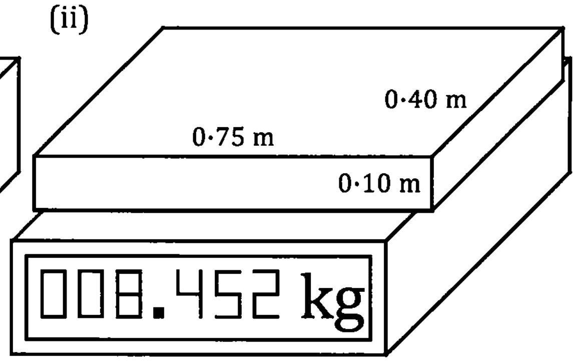


    (b) Charged at $8·40 for the first kilogram plus $5·20 per kilogram, or part thereof, after that.

    (i)



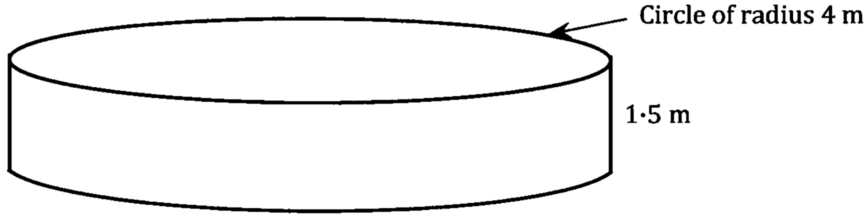
    (ii)



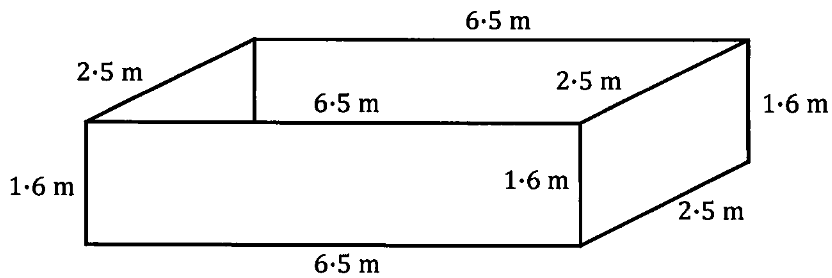
11. A company specializing in pool restoration equipment sells "epoxy pool coating" that can be painted onto the interior walls and floor of swimming pools.
For each of the pools shown below determine
(i) the <u>total</u> surface area of the interior walls and floor.
(ii) the amount of paint required, rounded up to the next whole litre, if three coats are required to all interior surfaces and each 1 litre of the paint will put one coat on an area of 8 square metres.

Note • Dimensions shown should be taken as interior dimensions.
• The open top surface of each of the pools shown is horizontal with the first being circular and the other three rectangular.

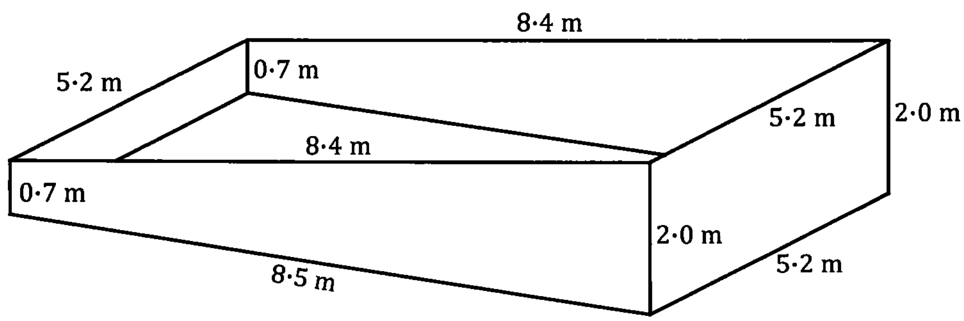
(a)



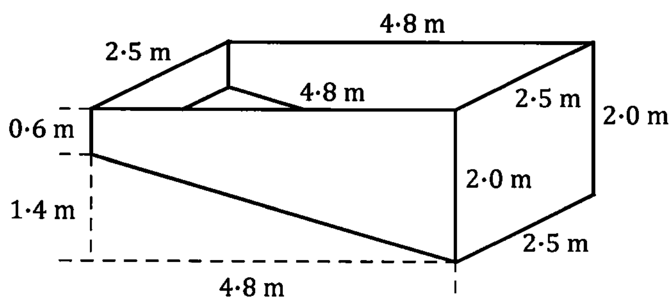
(b)



(c)

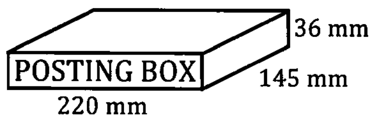


(d)

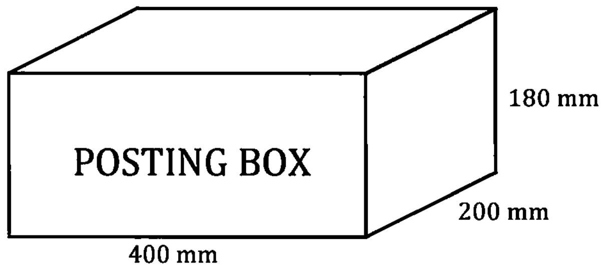


12. A company makes rectangular posting boxes using strong, lightweight card. Assuming the thickness of the card is 4 mm find the capacity of each of the following boxes, giving your answers rounded down to a whole multiple of five cubic centimetres. (The diagrams show *external* dimensions.)
(Assume each side, base and top are one thickness of card.)

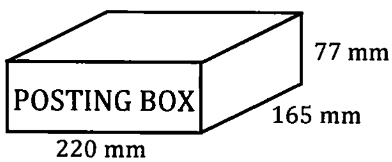
(a)



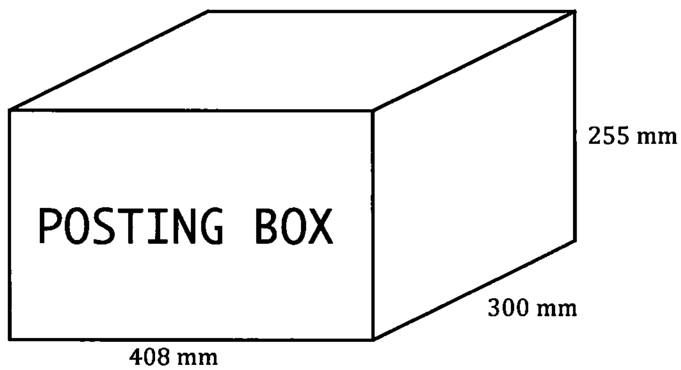
(b)



(c)



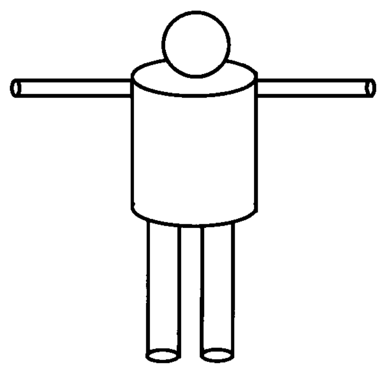
(d)



Modelling yourself very simply as two cylindrical arms, two cylindrical legs, a cylindrical body and a spherical head, estimate your surface area.

Compare your answer with that given by Mosteller's formula which claims to give a reasonable estimate of the surface area of a person of weight W kg and height h cm using the rule:

$$\text{Estimate of surface area in square metres} = \sqrt{\frac{W \times h}{3600}}$$



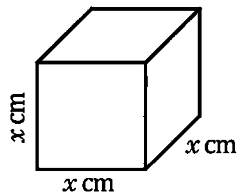
Research: As mentioned earlier, all of the prisms encountered in this chapter have been assumed to be "right" prisms. What is an "oblique" prism and how is its volume determined?

Inverse questions.

The following example, and the exercise that follows, again involves surface area or volume but now you are given one of these quantities for an object and your task is to determine an unknown length. (Indeed the last part of question 7 in the previous exercise was a simple one of this type.) As with the similar exercise in the previous chapter these "inverse questions" do require some equation solving ability and could be regarded as being beyond the requirements of the syllabus for this unit. I leave it to the reader and to teachers to decide whether to cover them or not.

Example 6

A solid cube has a total surface area of 384 cm^2. Determine the length of each edge of the cube.



Let each edge of the cube be x cm.

Each face will then have an area of x^2 cm^2.

Hence $\qquad 6x^2 = 384$

Divide by 6: $\qquad x^2 = 64$

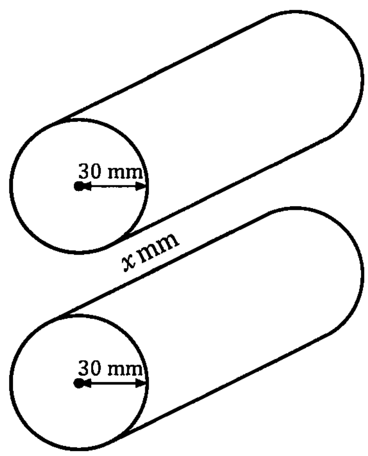
Solving and rejecting the negative solution gives $x = 8$.

Each edge of the cube is of length 8 cm.

Example 7

A solid cylinder with circular ends of radius 30 mm has a total surface area of 52 800 mm^2, to the nearest 100 mm^2.

Find the length of the cylinder giving your answer to the nearest millimetre.



Let the length of the cylinder be x mm.

Area of the two circles: $\qquad 2 \times \pi \times 30^2$ mm^2

Area of curved surface: $\qquad 2 \times \pi \times 30 \times x$ mm^2

Hence: $\qquad 2 \times \pi \times 30^2 + 2 \times \pi \times 30 \times x = 52800$

Take 1800π from each side: $\qquad 60\pi x = 52800 - 1800\pi$

Divide each side by 60π: $\qquad x = \dfrac{52800 - 1800\pi}{60\pi}$

$= 250$ mm to the nearest millimetre.

```
solve(2•π•30²+60•π•x=52800, x)
                {x=250.1126998}
```

```
52800−2×π×30²
                47145.13322
Ans÷(2×π×30)
                250.1126998
```

Example 8

A hemispherical bowl can hold 0·5 litres of liquid when filled level to the brim. Find the radius of the hemisphere.

If the radius of the hemisphere is r cm then volume = $\frac{2}{3}\pi r^3$ cm^3

$$\therefore \quad \frac{2}{3}\pi r^3 = 500$$

We solve this equation either by using the solve facility of some calculators, as shown on the right, or by algebraic manipulation as shown below.

solve$\left(\frac{2}{3}\bullet\pi\bullet x^3=500, x\right)$
{x=6.203504909}

× by 3 and ÷ by 2π $\quad r^3 = \dfrac{500\times3}{2\pi}$

Hence $\quad r = \sqrt[3]{\dfrac{500\times3}{2\pi}}$

$\quad\quad \approx 6{\cdot}204$

The radius of the hemisphere is 62 mm, to the nearest millimetre.

Exercise 9D.

1. A cube has a volume of 125 cm^3. Find (a) the length of each side of the cube,
(b) the surface area of the cube.

2. A cube has a surface area of 96 cm^2. Find (a) the length of each side of the cube,
(b) the volume of the cube.

3. Find, to the nearest millimetre, the radius of a sphere given that it has a volume of 74 cm^3.

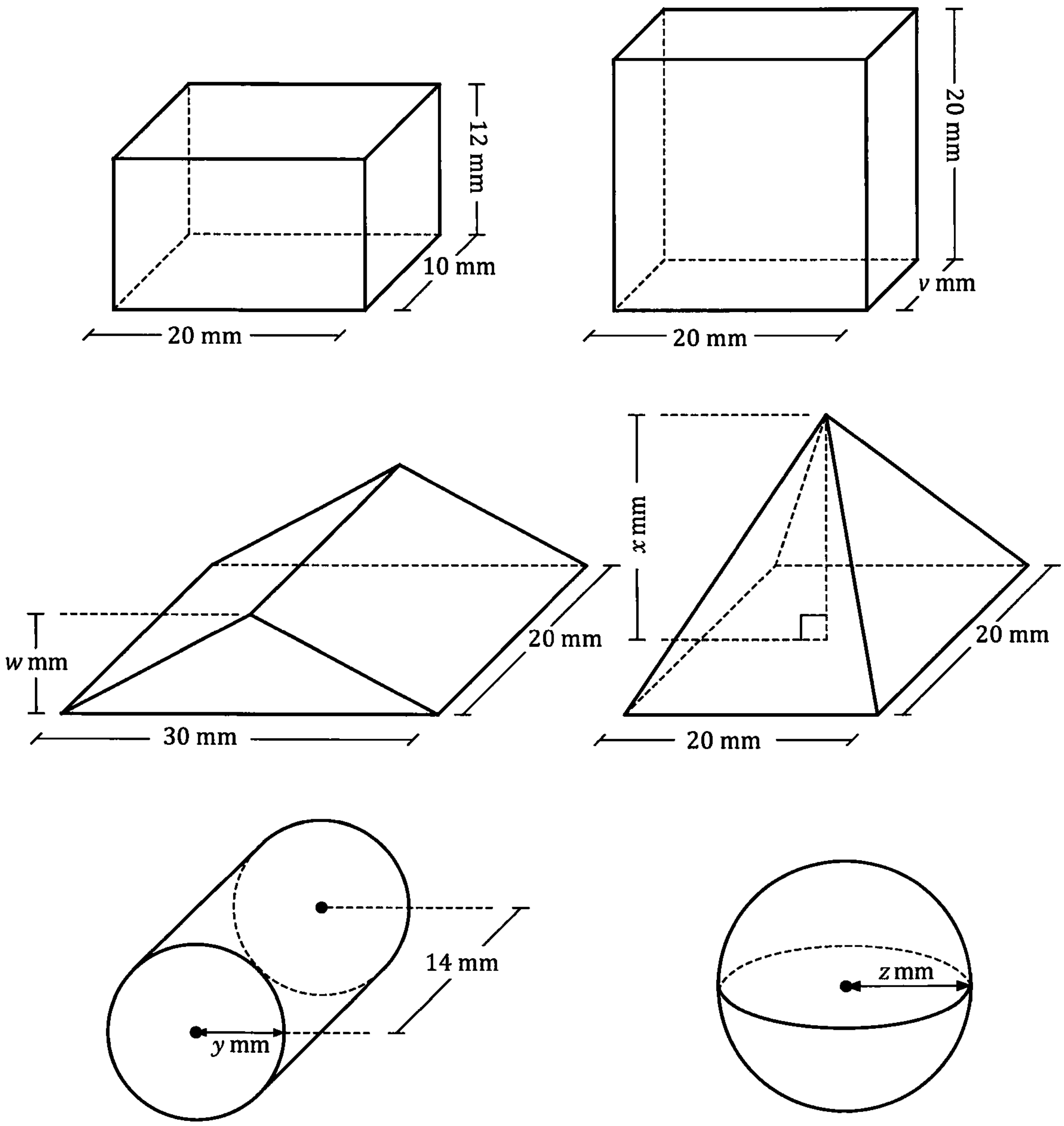
4. A hemispherical bowl can hold 1·25 litres of liquid when filled level to the brim. Find the radius of the hemisphere giving your answer to the nearest millimetre.

5. A solid sphere has a surface area of 7660 cm^2. Find the radius of the sphere giving your answer to the nearest millimetre.

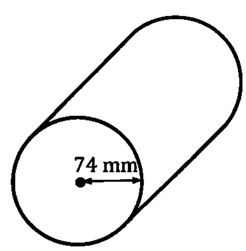
6. A cube has a volume of 3375 cm^3. Find the surface area of the cube.

7. A sphere has a volume of 3375 cm^3. Find the surface area of the sphere.

8. Each of the following solids have the same volume. Find the values of v, w, x, y and z. (For y and z give your answer correct to one decimal place.)

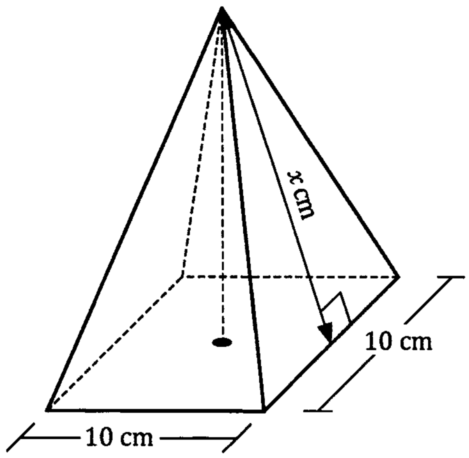


9. A solid cylinder with circular ends each of radius 74 mm has a total surface area of 133 900 mm^2, to the nearest square millimetre.
Find the length of the cylinder giving your answer to the nearest millimetre.

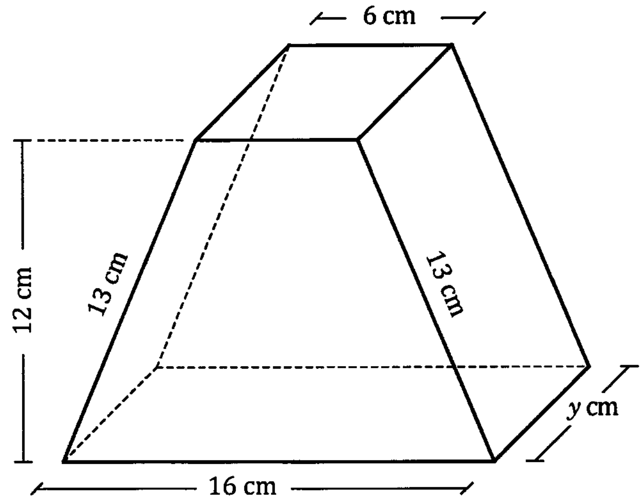


10. Each of the two solids shown below have the same surface area as a solid cube of side 10 cm. Find the values of x and y.

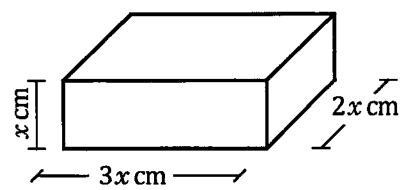
Square pyramid with top point vertically above centre of base.



Trapezoidal prism.



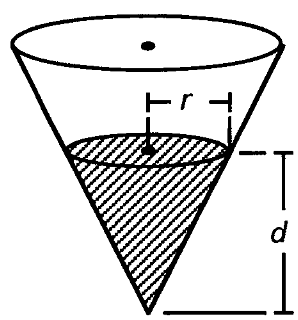
11. Given that the solid rectangular prism shown on the right has a total surface area of 137·5 cm^2 find the value of x.



12. A solid metal sphere of radius 20 cm is recast into two identical smaller spheres. To the nearest millimetre, and assuming no loss of metal in the process, what will be the radius of each of these two spheres?
How does the total surface area of the two smaller spheres compare with the surface area of the original sphere?

13. A solid metal sphere of radius 21·0 cm is to be melted down and recast into two cylinders, each with end radius of 14·0 cm, but with one cylinder twice the length of the other. Assuming no loss of metal in the process what will be the length of each cylinder?
How does the total surface area of the two cylinders compare with the surface area of the original sphere?

14. A container is in the shape of an upturned cone.
As liquid is poured into this container the ratio of the radius of the curved surface of the liquid to the depth of the liquid is always 1 : 2. I.e. for the situation shown on the right $r : d = 1 : 2$
What is the depth of the liquid (d in the diagram on the right) when the container holds 0·5 litres of liquid?



Miscellaneous Exercise Nine.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary section at the beginning of the book.

1. Each part of this question involves a student working out an answer which is marked and commented on by their teacher. For each situation write some explanation to the student of what it is they are doing wrong, what they should do to get the answer right and include some examples that show correct responses.

(a) Jamie is asked to work out

$$856\,000 \times 5\,200\,000.$$

Jamie uses his calculator and his written answer and his teacher's comment are shown below:

856000 × 5200000
4.4512E+12

$856\,000 \times 5\,200\,000 = 4{\cdot}4512\text{E}{+}12$ ✗

We do not write our answer in this way Jamie.

(b) Shania is asked to find the decimal equivalent of five sevenths.
Shania uses her calculator and her written answer and her teacher's comment are shown below:

5 ÷ 7
0.7142857143

$5 \div 7 = 0{\cdot}714\,285\,714\,3$ ✗

This is an approximation of the decimal equivalent Shani

(c) Jin is asked to work out, to the nearest dollar, the total cost of 15 identical items if 28 of these items cost a total of \$623.
Jin's written answer and the teacher's comment are shown below:

623÷28
22.25
15×22
330

\$623 ÷ 28 = \$22 to nearest \$1. 15 × \$22 = \$330
Fifteen of the items would cost \$330, to the nearest dollar. ✗

Avoid premature rounding Jin.

(d) Junior is asked: If a 433 cm length of wood is to be hand sawn into 7 pieces of equal length how long will each piece be? Junior's answer and his teacher's comment are shown below:

433 ÷ 7
61.85714286

Each piece will be 61·857 142 86 cm long. ✗

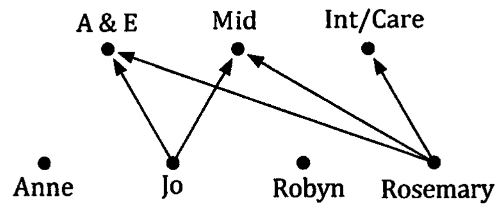
Inappropriate accuracy claimed Junior.

2. After 10% Goods and Sevices Tax (GST) has been added the total cost of an item is \$129·14. What was the cost of the item before GST was added?

3. Anne's row of the matrix on the right indicates that she is qualified to nurse in *Accident & Emergency* (A&E), and in *Intensive Care* (as indicated by the 1's) but not in *Midwifery* (indicated by 0).

	A & E	Mid	Int/Care
Anne	1	0	1
Jo			
Robyn	0	0	1
Rosemary			

The lines from the name Jo in the diagram indicate that Jo is qualified to work in *A&E* and *Midwifery* but not *Intensive Care.*

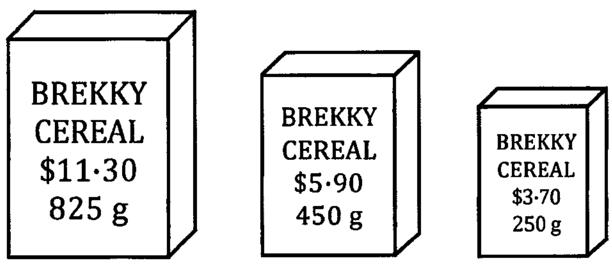


Complete both the matrix and the diagram so that they are consistent with each other.

4. Evaluating $2 + 3 \times 5^2$ gives an answer of 77.
Insert brackets to make each of the following statements true:
(a) $2 + 3 \times 5^2 = 227$ (b) $2 + 3 \times 5^2 = 289$ (c) $2 + 3 \times 5^2 = 125$

5. If $A = \begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$ determine BA + 3C.

6. Determine the best buy for the breakfast cereal deals shown on the right. (Each deal involves the same type of cereal.)



7. List the following in order, greatest interest earned first, and state the interest earned in each case.
\$5000 invested for 5 years at 8·5% per annum simple interest.
\$5000 invested for 5 years at 8% per annum compounded every 3 months.
\$5000 invested for 5 years at 8·2% per annum compounded every 6 months.

8. Assuming the earth to be a sphere of radius $6{\cdot}37 \times 10^3$ km find the surface area of the earth.
Approximately 70% of the earth's surface is covered by water. How many square kilometres of the earth surface is covered in water?

9. Copy and complete the following table:

	Unit cost	Number ordered	Sub total	Less 20% discount	Plus 10% tax
e.g.	*\$34·50*	*17*	*\$586·50*	*\$469·20*	*\$516·12*
	\$8·50	23			
	\$145·50		\$1891·50		
		56		\$358·40	
		8			\$7208·96

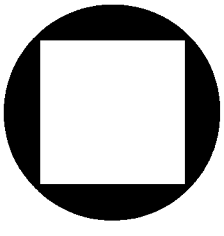
10. (a) What percentage profit is made when an item that cost \$247 is sold for \$328. (Give your answer to the nearest percent.)
(b) What must an item that cost \$24 be sold for to make a profit of at least 30%?
(c) An item sold for \$2842 gave the seller a profit of 16% on what he purchased the item for. How much did the seller purchase it for?
(d) An item sells at auction for \$4480. After the auctioneer's commission of 12% is taken from this amount, what remains gives the person who put the item into the auction a profit of 28% on what they purchased the item for. How much did the person putting the item into the auction pay for the item originally?

11. If $A = \begin{bmatrix} -2 & -1 & 2 \\ 1 & -1 & 3 \\ 3 & 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 & -1 \\ 11 & -2 & 8 \\ 5 & 1 & 3 \end{bmatrix}$ determine AB, without the assistance of your calculator.

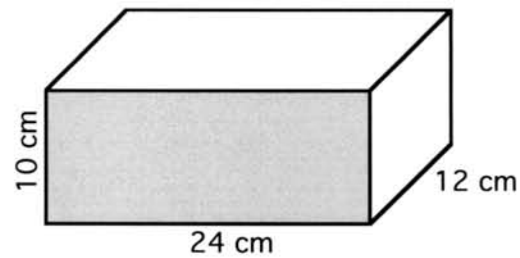
12. A square piece of glass has a diagonal of length 150 cm.
Find the area of the square.

13. A rectangular picture frame is twice as long as it is wide and its diagonals are each of length 30 cm.
Find the area of the rectangle.

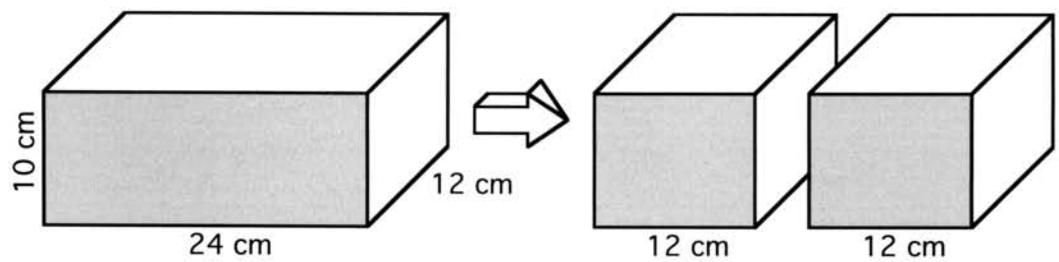
14. What percentage of the circle shown on the right is shaded?
Give your answer to the nearest 1%.
(The shape consists of a square drawn in a circle with all four vertices of the square just touching the circumference of the circle.)



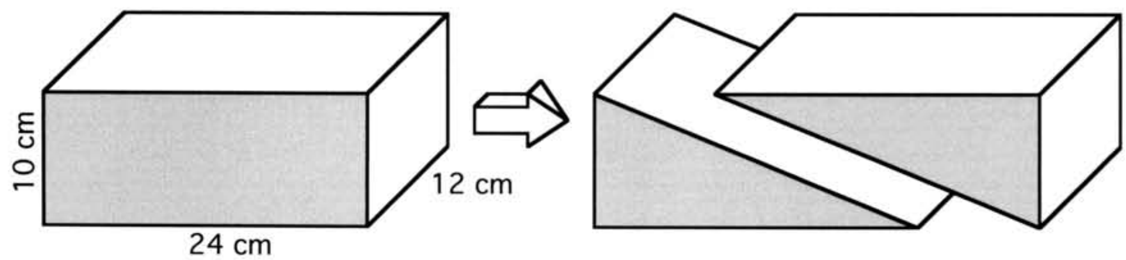
15. The rectangular prism shown on the right is to be cut into two equal pieces. Find the new total surface area, as a percentage of the total surface area of the original prism (to nearest percent) in each of the following situations:



(a)



(b)



16. Earlier in this book it was stated that the curved surface area of a cone of base radius r and slant height ℓ is given by the rule:

$$\text{Curved surface area} = \pi r \ell.$$

By considering this curved surface to be formed from a sector of a circle of radius ℓ, as shown below left, try to prove the above rule correct.