NELSON QMATHS 12 MATHEMATICAL METHODS

WORKED SOLUTIONS

Chapter 1 Logarithmic functions

Exercise 1.01 Logarithm laws

Question 1

a $3^4 = 81$

$\log_3 (81) = 4$

b $16^{\frac{1}{2}} = 4$

$\log_{16} (4) = \frac{1}{2}$

c $10^{-4} = 0.0001$

$\log_{10} (0.0001) = -4$

d $27^{\frac{2}{3}} = 9$

$\log_{27} (9) = \frac{2}{3}$

Question 2

a $\log_4 (64) = 3$

$4^3 = 64$

b $\log_{27} (3) = \dfrac{1}{3}$

$27^{\frac{1}{3}} = 3$

c $\log (0.01) = -2$

$10^{-2} = 0.01$

d $\log_8 (4) = \dfrac{2}{3}$

$8^{\frac{2}{3}} = 4$

Question 3

a $\log_5 (1) = \log_5 (5^0) = 0 \log_5 (5) = 0 \times 1 = 0$

b $\log_2 (0.5) = \log_2 \left(\dfrac{1}{2}\right) = \log_2 (2^{-1}) = -1 \times 1 = -1$

c $2 \log_3 (1) = 2 \log_3 (3^0) = 0 \times 2 \times 1 = 0$

d $\log_a (1), a > 0 = \log_a (a^0) = 0 \times 1 = 0$

e $\log_5 (5) = \log_5 (5^1) = 1 \times \log_5 (5) = 1 \times 1 = 1$

f $[\log_4 (4)]^2 = [\log_4 (4^1)]^2 = [1 \times \log_4 (4)]^2 = [1 \times 1]^2 = 1^2 = 1$

g $\log_2 (8) = \log_2 (2^3) = 3 \log_2 (2) = 3 \times 1 = 3$

h $\log_8 (2) = \log_8 (8^{\frac{1}{3}}) = \dfrac{1}{3} \times \log_8 (8) = \dfrac{1}{3} \times 1 = \dfrac{1}{3}$

Question 4

a $\log_4(2) + \log_4(64) = \log_4(2 \times 64)$

$= \log_4(128)$

$= \log_4\left(4^{\frac{7}{2}}\right)$

$= \frac{7}{2}\log_4(4)$

$= \frac{7}{2} \times 1$

$= 3.5$

b $\log_9(81) - \log_9(3) = \log_9\left(9^2\right) - \log_9\left(9^{\frac{1}{2}}\right)$

$= 2\log_9(9) - \frac{1}{2}\log_9(9)$

$= 2 - \frac{1}{2}$

$= 1\frac{1}{2}$ or 1.5

c $\log_2(10) + 2\log_2(4) - \log_2(80) = \log_2(10) + \log_2\left(4^2\right) - \log_2(80)$

$= \log_2\left(\frac{10 \times 16}{80}\right)$

$= \log_2(2)$

$= 1$

d $\log_7(\log_5(5)) = \log_7(1) = \log_7\left(7^0\right) = 0 \times \log_7(7) = 0$

e $\log_7\left(\frac{5}{11}\right) + 2\log_7\left(\frac{1}{4}\right) - \log_7\left(\frac{49}{11}\right) - \log_7\left(\frac{5}{16}\right)$

$= \log_7\left(\frac{5}{11}\right) + \log_7\left(\left(\frac{1}{4}\right)^2\right) - \{\log_7\left(\frac{49}{11}\right) + \log_7\left(\frac{5}{16}\right)\}$

$= \log_7\left(\frac{5}{11}\times\frac{1}{16}\div\left(\frac{49}{11}\times\frac{5}{16}\right)\right)$

$= \log_7\left(\frac{1}{49}\right)$

$= \log_7\left(7^{-2}\right)$

$= -2 \times \log_7 (7)$

$= -2$

f $\log_2\left(\sqrt[5]{\frac{1}{1024}}\right) \quad = \frac{1}{5}\log_2\left(\frac{1}{1024}\right)$

$= \frac{1}{5}\log_2\left(\frac{1}{2^{10}}\right)$

$= \frac{1}{5}\log_2\left(2^{-10}\right)$

$= -10 \times \frac{1}{5}\log_2(2)$

$= -2$

Question 5

a $\log_6 (7y^2) = \log_6 (7) + \log_6 (y^2)$

$= \log_6 (7) + 2\log_6 (y)$

b $\log_3 (5xz^3) = \log_3 (5) + \log_3 (x) + \log_3 (z^3)$

$= \log_3 (5) + \log_3 (x) + 3\log_3 (z)$

c $\log_4\left(\frac{1}{w^5}\right) = \log_4\left(w^{-5}\right)$

$= -5\log_4(w)$

d $\log_8\left(\frac{2x}{y^2z^3}\right) = \log_8(2) + \log_8(x) - \log_8\left(y^2\right) - \log_8\left(z^3\right)$

$= \log_8\left(8^{\frac{1}{3}}\right) + \log_8(x) - 2\log_8(y) - 3\log_8(z)$

$= \frac{1}{3}\log_8(8) + \log_8(x) - 2\log_8(y) - 3\log_8(z)$

$= \frac{1}{3} + \log_8(x) - 2\log_8(y) - 3\log_8(z)$

e $\log_7(x^2 - 4) = \log_7[(x + 2)(x - 2)] = \log_7(x + 2) + \log_7(x - 2)$

f $\log_5\left(\frac{y^3(y+3)}{(y-2)^2}\right) = \log_5\left(y^3\right) + \log_5(y+3) - \log_5(y-2)^2$

$= 3\log_5(y) + \log_5(y+3) - 2\log_5(y-2)$

g $\log_2\left(\sqrt{\frac{x^2+y^2}{xy}}\right) = \log_2\left(\left(\frac{x^2+y^2}{xy}\right)^{\frac{1}{2}}\right)$

$= \frac{1}{2}\log_2\left(\frac{x^2+y^2}{xy}\right)$

$= \frac{1}{2}\left[\log_2\left(x^2+y^2\right) - \log_2(xy)\right]$

$= \frac{1}{2}\left[\log_2\left(x^2+y^2\right) - \log_2(x) - \log_2(y)\right]$

h $\log_3\left(\frac{5}{\sqrt{z^2+1}}\right) = \log_3\left(5\left(z^2+1\right)^{-\frac{1}{2}}\right)$

$= \log_3(5) + \log_3\left(z^2+1\right)^{-\frac{1}{2}}$

$= \log_3(5) - \frac{1}{2}\log_3\left(z^2+1\right)$

Question 6

a $5\log_2(x) + 3\log_2(y)$

$= \log_2\left(x^5\right) + \log_2\left(y^3\right)$

$= \log_2\left(x^5y^3\right)$

b $2\log_6(p) - \log_6(11) - \log_6(m)$

$= \log_6\left(p^2\right) - \log_6(11) - \log_6(m)$

$= \log_6\left(\frac{p^2}{11m}\right)$

c $2[\log(y) + \log(y+2) - \log(y^2-1)]$

$= 2\left[\log\left(\frac{y(y+2)}{\left(y^2-1\right)}\right)\right]$

$= \log\left(\frac{y(y+2)}{y^2-1}\right)^2$

d $4[3\log_4(x) - \log_4(x+1) - \log_4(x^2-3)]$

$= 4[\log_4(x)^3 - \log_4(x+1) - \log_4(x^2-3)]^4$

$= 4\left[\log_4\left(\frac{x^3}{(x+1)\left(x^2-3\right)}\right)\right]$

$= \log_4\left(\frac{x^3}{(x+1)\left(x^2-3\right)}\right)^4$

e $\frac{1}{2}\log_5(17) - 3\log_5(x) - 4\log_5(y)$

$= \log_5(17)^{\frac{1}{2}} - \log_5(x)^3 - \log_5(y)^4$

$= \log_5\left(\frac{\sqrt{17}}{x^3y^4}\right)$

f $\frac{1}{3}\log_7(x-3) + 3\log_7(x) + 2\log_7(3+x)$

$= \log_7(x-3)^{\frac{1}{3}} + \log_7(x)^3 + \log_7(3+x)^2$

$= \log_7\left(x^3(3+x)^2\sqrt[3]{x-3}\right)$

g $\frac{1}{2}[\log_3(x) + 3\log_3(y) - 5\log_3(x-2)]$

$= \frac{1}{2}\left[\log_3(x) + \log_3(y^3) - \log_3(x-2)^5\right]$

$= \frac{1}{2}\left[\log_3\frac{xy^3}{(x-2)^5}\right]$

$= \left[\log_3\frac{xy^3}{(x-2)^5}\right]^{\frac{1}{2}}$

$= \left[\log_3\sqrt{\frac{xy^3}{(x-2)^5}}\right]$

h $\frac{1}{2}\log_2(x+1) + \frac{1}{2}\log_2(x-1) + \log_2(5)$

$= \log_2\left(\sqrt{x+1}\right) + \log_2\left(\sqrt{x-1}\right) + \log_2(5)$

$= \log_2\left(5\sqrt{(x+1)(x-1)}\right)$

Question 7

a $a^{\log_a(x^2-3)} = x^2 - 3$

b $25^{\log_5 8} = 5^{2(\log_5 8)}$

$= 5^{(\log_5 8^2)}$

$= 5^{\log_5 64}$

$= 64$

c $10^{\frac{1}{2}\log 7} = 10^{\log\sqrt{7}}$

$= \sqrt{7}$

d $4^{3\log_4 6} = 4^{\log_4 6^3}$

$= 6^3$

$= 216$

e $3^{-2\log_3 7} = 3^{\log_3 7^{-2}}$

$= 3^{\log_3 \frac{1}{49}}$

$= \frac{1}{49}$

Question 8

a

$\log_2(36)$

$= \frac{\log 36}{\log 2}$

$= \frac{1.55630...}{0.30102...}$

$= 5.169925...$

≈ 5.1699

b

$$\log_{100}(500)$$
$$= \frac{\log 500}{\log 100}$$
$$= \frac{2.69897}{2}$$
$$= 1.349485...$$
$$\approx 1.3495$$

c

$$\log_{0.2}(21)$$
$$= \frac{\log 21}{\log 0.2}$$
$$= \frac{1.32221...}{-0.69897...}$$
$$= -1.89166...$$
$$\approx -1.8917$$

d

$$\log_{24}(3.18)$$
$$= \frac{\log 24}{\log(3.18)}$$
$$= \frac{0.502427...}{1.380211...}$$
$$= 0.364021...$$
$$\approx 0.3640$$

Question 9

a

$$\log_3 81$$

$$= \frac{\log 81}{\log 3}$$

$$= \frac{\log 3^4}{\log 3}$$

$$= \frac{4\log 3}{\log 3}$$

$$= 4$$

b

$$\log_9 27$$

$$= \frac{\log 27}{\log 9}$$

$$= \frac{\log 3^3}{\log 3^2}$$

$$= \frac{3\log 3}{2\log 3}$$

$$= \frac{3}{2}$$

Question 10

a $\log_a 30$

$$= \log_a (2\times 3\times 5)$$

$$= \log_a 2 + \log_a 3 + \log_a 5$$

$$= 0.67 + 1.07 + 1.56$$

$$= 3.30$$

b $\log_a (40)$

$= \log_a (5\times 8)$

$= \log_a (5\times 2^3)$

$= \log_a 5 + 3\log_a 2$

$= 1.56 + 3\times 0.67$

$= 3.57$

c $\log_a \left(\frac{3}{10}\right)$

$= \log_a \left(\frac{3}{2\times 5}\right)$

$= \log_a 3 - \log_a 2 - \log_a 5$

$= 1.07 - 0.67 - 1.56$

$= -1.16$

d $\log_a \left(\frac{1}{\sqrt{15}}\right)$

$= \log_a (15)^{-\frac{1}{2}}$

$= -\frac{1}{2}\log_a (15)$

$= -\frac{1}{2}\log_a (3\times 5)$

$= -\frac{1}{2}\left[\log_a 3 + \log_a 5\right]$

$= -\frac{1}{2}\left[1.07 + 1.56\right]$

-1.32

e $\log_a \left(-\frac{3}{5}\right)$

$= \log_a (-3) - \log_a 5$

$\log_a (-3)$ is undefined $\therefore$ no solution

Question 11

a

If $n = \log_3 x$ then $3^n = x$

$$3x = 3 \times x$$
$$= 3 \times 3^n$$
$$= 3^{n+1}$$

b $\log_3\left(3x^4\right)$

$$= \log_3 3 + \log_3 x^4$$
$$= 1 + 4\log_3 x$$
$$= 1 + 4n$$

c $\log_x\left(243\right)$

$$= \log_x\left(3^5\right)$$
$$= 5\log_x 3$$
$$= 5\left(\frac{\log_3 3}{\log_3 x}\right)$$
$$= 5 \times \frac{1}{n}$$
$$= \frac{5}{n}$$

d $\log_3\left(\frac{x^2}{27}\right)$

$$= \log_3\left(x^2\right) - \log_3 27$$
$$= 2\log_3 x - \log_3 27$$
$$= 2n - \log_3\left(3^3\right)$$
$$= 2n - 3\log_3 3$$
$$= 2n - 3$$

Question 12

a

$$\log y = 2\log(4) - 5\log(x)$$
$$= \log(4^2) - \log(x^5)$$
$$= \log\left(\frac{16}{x^5}\right)$$
$$y = \frac{16}{x^5}$$

b

$$\log_3(y) = 4\log_3(x) - 2$$
$$= \log_3(x^4) - 2\log_3 3$$
$$= \log_3(x^4) - \log_3(3^2)$$
$$= \log_3\left(\frac{x^4}{9}\right)$$
$$y = \frac{x^4}{9}$$

c

$$\log_4(2xy) = 2.5$$
$$4^{2.5} = 2xy$$
$$(2^2)^{\frac{5}{2}} = 2xy$$
$$32 = 2xy$$
$$y = \frac{16}{x}$$

d

$$\log_3\left(\frac{2x}{y}\right)+3=\log_3(2)$$

$$\log_3(2x)-\log_3(y)+3=\log_3(2)$$

$$\log_3(y)=\log_3(2x)+3\log_3(3)-\log_3(2)$$

$$=\log_3(2x)+\log_3\left(3^3\right)-\log_3(2)$$

$$=\log_3\left(\frac{54x}{2}\right)$$

$$y=27x$$

Question 13

$$\log_m(15)+\log_m(x)=0$$

$$\log_m(15x)=\log_m(1)$$

$$15x=1$$

$$x=\frac{1}{15}$$

Question 14

$$\log_b(a)$$

$$=\frac{\log_{10}(a)}{\log_{10}(b)}$$

$$=\frac{1}{\frac{\log_{10}(b)}{\log_{10}(a)}}$$

$$=\frac{1}{\log_a(b)}$$

Question 15

$\log_a\left(x^n\right)$

$=\log_a\left(x\times x\times x\times ....\right), n \text{ times}$

$=\log_a\left(x\right)+\log_a\left(x\right)+\log_a\left(x\right)+...$

$=n\log_a\left(x\right)$

Question 16

a $\log_n\left(xy^2\right)$

$=\log_n(x)+\log_n\left(y^2\right)$

$=\log_n(x)+2\log_n(y)$

$=3+2\times 5$

$=13$

b $\log_n\left(nx^3y\right)$

$=\log_n\left(n\right)+\log_n\left(x^3\right)+\log_n\left(y\right)$

$=\log_n\left(n\right)+3\log_n\left(x\right)+\log_n\left(y\right)$

$=1+3\times 3+5$

$=15$

c $\log_n\left(\dfrac{x}{\sqrt{y}}\right)$

$=\log_n\left(x\right)-\log_n\left(y^{\frac{1}{2}}\right)$

$=\log_n\left(x\right)-\dfrac{1}{2}\log_n\left(y\right)$

$=3-\dfrac{1}{2}\times 5$

$=0.5$

d $\log_n\left(\sqrt[3]{xy^3}\right)$

$$= \log_n\left(xy^3\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\left[\log_n(x) + \log_n\left(y^3\right)\right]$$

$$= \frac{1}{3}\left[\log_n(x) + 3\log_n(y)\right]$$

$$= \frac{1}{3}[3 + 3 \times 5]$$

$$= 6$$

Exercise 1.02 Indicial equations

Question 1

a $6^x = 1$

$6^x = 6^0$

$x = 0$

b $9^k = 27$

$(3^2)^k = 3^3$

$2k = 3$

$k = \frac{3}{2}$

c $4^x = 16\sqrt{2}$

$(2^2)^x = 2^4 \times 2^{\frac{1}{2}}$

$2^{2x} = 2^{\frac{9}{2}}$

$2x = \frac{9}{2}$

$x = \frac{9}{4}$

d $8^{-r} = \frac{1}{2}$

$(2^3)^{-r} = 2^{-1}$

$-3r = -1$

$r = \frac{1}{3}$

e $4^{2x} = 64$

$4^{2x} = 4^3$

$2x = 3$

$x = \frac{3}{2}$

f $4^y = \sqrt{2}$

$\left(2^2\right)^y = 2^{\frac{1}{2}}$

$2y = \frac{1}{2}$

$y = \frac{1}{4}$

g $3^{\frac{x}{2}} = 27$

$3^{\frac{x}{2}} = 3^3$

$\frac{x}{2} = 3$

$x = 6$

h $\frac{1}{27^g} = \sqrt{3}$

$\left(3^{3g}\right)^{-1} = 3^{\frac{1}{2}}$

$-3g = \frac{1}{2}$

$g = -\frac{1}{6}$

Question 2

a $5^x = 5^{7x-2}$

$$x = 7x - 2$$

$$-6x = -2$$

$$x = \frac{1}{3}$$

b $7^{1-u} = 7^{-3}$

$$1 - u = -3$$

$$-u = -4$$

$$u = 4$$

c $5^{2k-5} = 1$

$$5^{2k-5} = 5^0$$

$$2k - 5 = 0$$

$$2k = 5$$

$$k = \frac{5}{2}$$

d $81^{1-2m} = 27$

$$3^{4(1-2m)} = 3^3$$

$$4 - 8m = 3$$

$$-8m = -1$$

$$m = \frac{1}{8}$$

e $4^{y^2} = 4^{6-y}$

$y^2 = 6 - y$

$y^2 + y - 6 = 0$

$(y + 3)(y - 2) = 0$

$y = -3, 2$

f $64^{x+1} = \sqrt{16}$

$4^{3(x+1)} = 4^{\frac{1}{2}}$

$3x + 3 = \frac{1}{2}$

$3x = -\frac{5}{2}$

$x = -\frac{5}{6}$

g $4^z = 16^{z+5}$

$4^z = 4^{2(z+5)}$

$z = 2z + 10$

$z = -10$

h $9^{5-9x} = \frac{1}{27^{x-2}}$

$\left(3^2\right)^{5-9x} = \frac{1}{3^{3(x-2)}}$

$3^{2(5-9x)} = 3^{-3(x-2)}$

$10 - 18x = -3x + 6$

$-15x = -4$

$x = \frac{4}{15}$

Question 3

a $4^x - 4^{x-1} = 3$

$$4^x - 4^x \times 4^{-1} = 3$$

$$\text{Let } a = 4^x$$

$$a - \frac{a}{4} = 3$$

$$4a - a = 12$$

$$3a = 12$$

$$a = 4$$

$$4^x = 4$$

$$x = 1$$

b $2^{m+4} - 2^m = 120$

$$2^4 \times 2^m - 2^m = 120$$

$$\text{Let } a = 2^m$$

$$16a - a = 120$$

$$15a = 120$$

$$a = 8$$

$$2^m = 8$$

$$m = 3$$

c $3^n - 3^{n-1} = 54$

$3^n - 3^n \times 3^{-1} = 54$

$\text{Let } a = 3^n$

$a - \frac{a}{3} = 54$

$3a - a = 162$

$2a = 162$

$a = 81$

$3^n = 3^4$

$n = 4$

d $2^x - 2^{-x} = \frac{3}{2}$

$2^x - \frac{1}{2^x} = \frac{3}{2}$

$\text{Let } a = 2^x$

$a - \frac{1}{a} = \frac{3}{2}$

$2a^2 - 2 = 3a$

$2a^2 - 3a - 2 = 0$

$(2a+1)(a-2) = 0$

$a = -\frac{1}{2} \quad \text{or} \quad a = 2$

$2^x = -2^{-1} \quad \text{or} \quad 2^x = 2^1$

$\text{no solution} \quad \text{or} \quad x = 1$

e $2^{y+6} - 9 \times 2^y = 110$

$$2^y \times 2^6 - 9 \times 2^y = 110$$

$$\text{Let } a = 2^y$$

$$64a - 9a = 110$$

$$55a = 110$$

$$a = 2$$

$$2^y = 2$$

$$y = 1$$

f $\dfrac{27^k - 27^{k-1}}{3^k} = 26$

$$\frac{3^{3k} - 3^{3(k-1)}}{3^k} = 26$$

$$\frac{3^{3k} - 3^{3k} \times 3^{-3}}{3^k} = 26$$

$$\text{Let } a = 3^k$$

$$\frac{a^3 - \frac{a^3}{27}}{a} = 26$$

$$\frac{a^{\not{3}2}\left(1 - \frac{1}{27}\right)}{\not{a}1} = 26$$

$$a^2 \times \frac{26}{27} = 26$$

$$a^2 = 27$$

$$\left(3^k\right)^2 = 3^3$$

$$2k = 3$$

$$k = \frac{3}{2}$$

Question 4

a $7^x = 12$

$$\log\left(7^x\right) = \log(12)$$

$$x\log(7) = \log(12)$$

$$x = \frac{\log(7)}{\log(12)}$$

$$x = 1.27698...$$

$$x \approx 1.277$$

b $12^{3x} = 38$

$$\log\left(12^{3x}\right) = \log(38)$$

$$3x\log(12) = \log(38)$$

$$3x = \frac{\log(38)}{\log(12)}$$

$$3x = 1.46387...$$

$$x = 0.48795...$$

$$x \approx 0.488$$

c $3^{4x + 1} = 17$

$$\log\left(3^{4x+1}\right) = \log(17)$$

$$(4x+1)\log(3) = \log(17)$$

$$4x+1 = \frac{\log(17)}{\log(3)}$$

$$4x+1 = 2.5789...$$

$$4x = 1.5789...$$

$$x = 0.39472...$$

$$x \approx 0.395$$

d $9^{x+2} = 4$

$$\log\left(9^{x+2}\right) = \log(4)$$

$$(x+2)\log(9) = \log(4)$$

$$x+2 = \frac{\log(4)}{\log(9)}$$

$$x+2 = 0.63092...$$

$$x = -1.36907...$$

$$x \approx -1.369$$

e $6^{x-1} = 4^{x}$

$$\log\left(6^{x-1}\right) = \log\left(4^{x}\right)$$

$$(x-1)\log(6) = x\log(4)$$

$$x\log(6) - \log(6) = x\log(4)$$

$$x\log(6) - x\log(4) = \log(6)$$

$$x\left[\log(6) - \log(4)\right] = \log(6)$$

$$x = \frac{\log(6)}{\log(6) - \log(4)}$$

$$x = 4.41902...$$

$$x \approx 4.419$$

f $5^{3x+2} - 4^x = 0$

$$5^{3x+2} = 4^x$$

$$\log\left(5^{3x+2}\right) = \log\left(4^x\right)$$

$$(3x+2)\log(5) = x\log(4)$$

$$3x\log(5) + 2\log(5) = x\log(4)$$

$$3x\log(5) - x\log(4) = -2\log(5)$$

$$x\left[3\log(5) - \log(4)\right] = -2\log(5)$$

$$x = \frac{-2\log(5)}{3\log(5) - \log(4)}$$

$$x = -0.93517...$$

$$x \approx -0.935$$

Question 5

a $5^{x-1} = 2^{x+1}$

$$\log\left(5^{x-1}\right) = \log\left(2^{x+1}\right)$$

$$(x-1)\log(5) = (x+1)\log(2)$$

$$x\log(5) - \log(5) = x\log(2) + \log(2)$$

$$x\left(\log(5) - \log(2)\right) = \log(2) + \log(5)$$

$$x = \frac{\log(2) + \log(5)}{\log(5) - \log(2)}$$

$$x = 2.5129...$$

$$x \approx 2.513$$

b $9^{2x+3} = 7^{5x-1}$

$$\log\left(9^{2x+3}\right) = \log\left(7^{5x-1}\right)$$

$$(2x+3)\log(9) = (5x-1)\log(7)$$

$$2x\log(9) + 3\log(9) = 5x\log(7) - \log(7)$$

$$x\left(2\log(9) - 5\log(7)\right) = -\log(7) - 3\log(9)$$

$$x = \frac{-\log(7) - 3\log(9)}{2\log(9) - 5\log(7)}$$

$$x = 1.60026...$$

$$x \approx 1.600$$

c $12^{2x+3} = 8^{3x-1}$

$$\log\left(12^{2x+3}\right) = \log\left(8^{3x-1}\right)$$

$$(2x+3)\log(12) = (3x-1)\log(8)$$

$$2x\log(12) + 3\log(12) = 3x\log(8) - \log(8)$$

$$x\left(2\log(12) - 3\log(8)\right) = -\log(8) - 3\log(12)$$

$$x = \frac{-\log(8) - 3\log(12)}{2\log(12) - 3\log(8)}$$

$$x = 7.51602...$$

$$x \approx 7.516$$

d $6^{2x + 5} = 11^{3x - 2}$

$\log\left(6^{2x+5}\right) = \log\left(11^{3x-2}\right)$

$(2x+5)\log(6) = (3x-2)\log(11)$

$2x\log(6) + 5\log(6) = 3x\log(11) - 2\log(11)$

$x\left(2\log(6) - 3\log(11)\right) = -2\log(11) - 5\log(6)$

$x = \dfrac{-2\log(11) - 5\log(6)}{2\log(6) - 3\log(11)}$

$x = 3.80995...$

$x \approx 3.810$

e $7^{x - 1} - 5^{2x + 1} = 0$

$7^{x-1} = 5^{2x+1}$

$\log\left(7^{x-1}\right) = \log\left(5^{2x+1}\right)$

$(x-1)\log(7) = (2x+1)\log(5)$

$x\log(7) - \log(7) = 2x\log(5) + \log(5)$

$x\left(\log(7) - 2\log(5)\right) = \log(5) + \log(7)$

$x = \dfrac{\log(5) + \log(7)}{\log(7) - 2\log(5)}$

$x = -2.79296...$

$x \approx -2.793$

f $21^{2x+1} - 15^{3x-1} = 0$

$\log\left(21^{2x+1}\right) = \log\left(15^{3x-1}\right)$

$(2x+1)\log(21) = (3x-1)\log(15)$

$2x\log(21) + \log(21) = 3x\log(15) - \log(15)$

$x\left(2\log(21) - 3\log(15)\right) = -\log(15) - \log(21)$

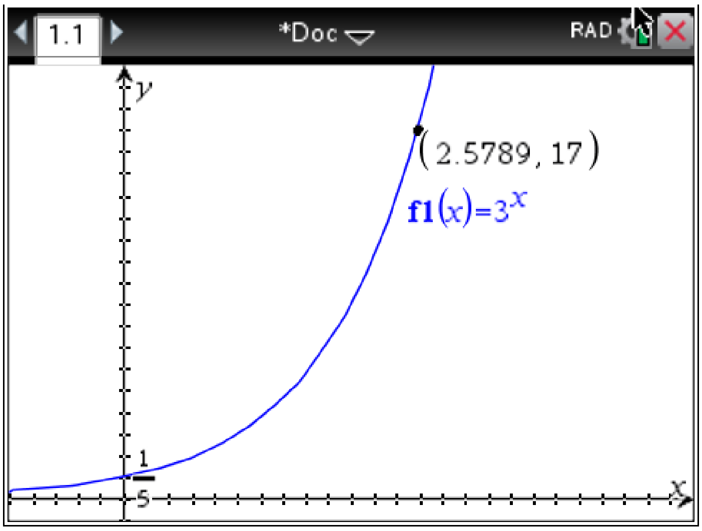
$x = \dfrac{-\log(15) - \log(21)}{2\log(21) - 3\log(15)}$

$x = 2.8266...$

$x \approx 2.827$

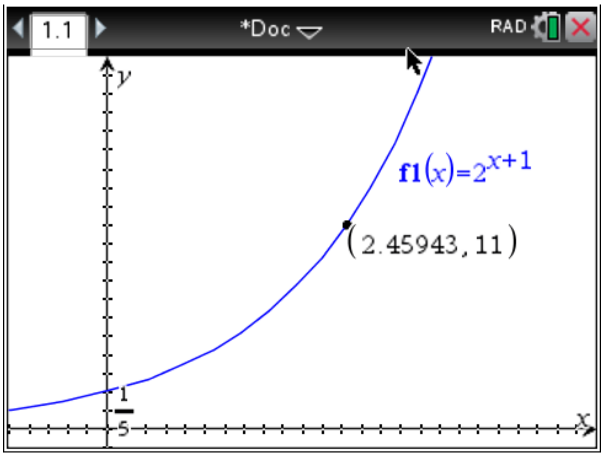
Question 6

a $3^x = 17$



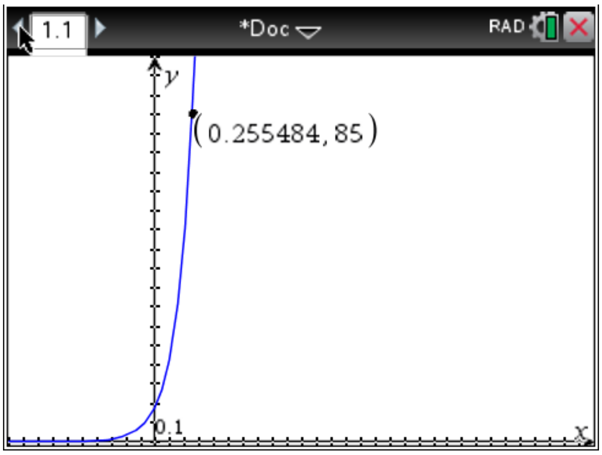
$x \approx 2.579$

b $2^{x+1} = 11$



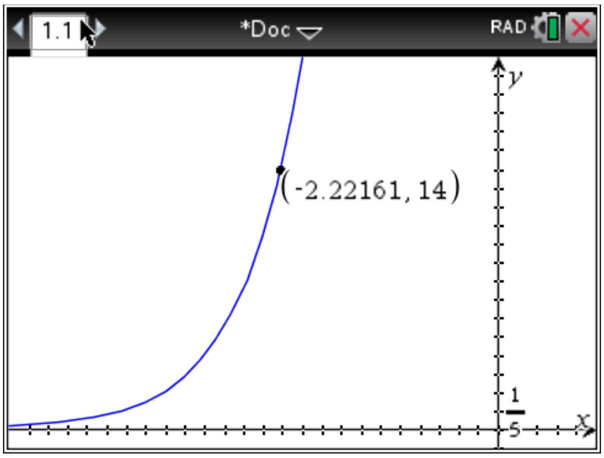
$x \approx 2.459$

c $9^{4x+1} = 85$



$x \approx 0.255$

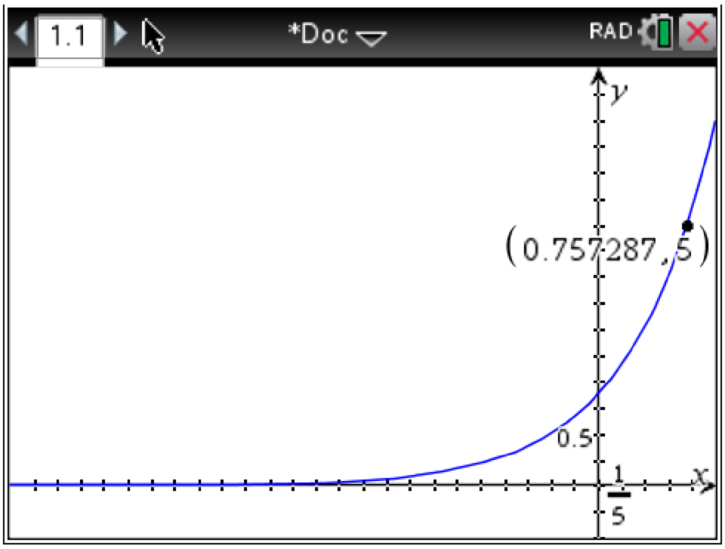
d $4 \times 5^{x+3} = 14$



$x \approx -2.222$

e $7 \times 4^{x-1} - 5 = 0$

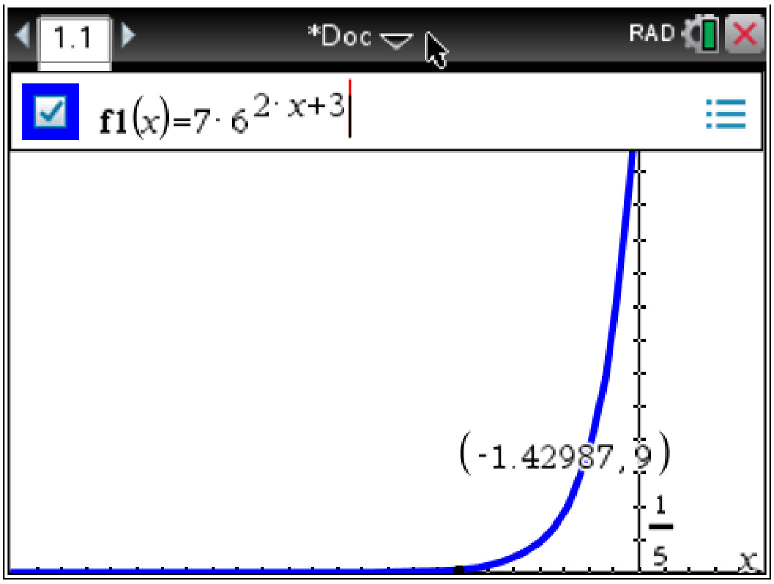
$7 \times 4^{x-1} = 5$



$x \approx 0.757$

f $7 \times 6^{2x+3} - 9 = 0$

$7 \times 6^{2x+3} = 9$



$x \approx -1.430$

Question 7

a $64 \times 16^{-3x} = 16^{3x-4}$

$$4^3 \times 4^{2(-3x)} = 4^{2(3x-4)}$$

$$4^3 \times 4^{-6x} = 4^{6x} \times 4^{-8}$$

$$\frac{4^3}{4^{6x}} = \frac{4^{6x}}{4^8}$$

$$4^{11} = 4^{12x}$$

$$12x = 11$$

$$x = \frac{11}{12}$$

b $243^{m+2} \times 9^{2m-3} = 9$

$$3^{5(m+2)} \times 3^{2(2m-3)} = 3^2$$

$$3^{5m} \times 3^{10} \times 3^{4m} \times 3^{-6} = 3^2$$

$$3^{9m} \times 3^4 = 3^2$$

$$3^{9m} = 3^{-2}$$

$$9m = -2$$

$$m = -\frac{2}{9}$$

c $\frac{81^{3n+2}}{243^{-n}} = 3^{4-n}$

$$\frac{3^{4(3n+2)}}{3^{-5n}} = 3^{4-n}$$

$$3^{17n} \times 3^8 = 3^4 \times 3^{-n}$$

$$3^{18n} = 3^{-4}$$

$$18n = -4$$

$$n = -\frac{2}{9}$$

d $16^{2k-3} \times 4^{-2k+1} = 32$

$$4^{2(2k-3)} \times 4^{-2k+1} = 2 \times 4^2$$

$$4^{4k} \times 4^{-6} \times 4^{-2k} \times 4 = 4^{\frac{1}{2}} \times 4^2$$

$$4^{2k} = \frac{4^{\frac{5}{2}}}{4^{-5}}$$

$$4^{2k} = 4^{\frac{15}{2}}$$

$$2k = \frac{15}{2}$$

$$k = \frac{15}{4} = 3\frac{3}{4}$$

e $6^{3b} \times 6^{-b} = 6^{-2b+1}$

$$6^{3b} \times 6^{-b} = 6^{-2b} \times 6$$

$$6^{2b} = 6^{-2b} \times 6$$

$$\frac{6^{2b}}{6^{-2b}} = 6$$

$$6^{4b} = 6^1$$

$$4b = 1$$

$$b = \frac{1}{4}$$

f $3^{-2x+3} \times 3^{-2x-5} = 3^{-x-1}$

$$3^{-2x} \times 3^3 \times 3^{-2x} \times 3^{-5} = 3^{-x} \times 3^{-1}$$

$$3^{-4x} \times 3^{-2} = 3^{-x} \times 3^{-1}$$

$$3^{-3x} = 3^1$$

$$-3x = 1$$

$$x = -\frac{1}{3}$$

Question 8

$2^{2m-2} - 2^{m-2} = 3$

$2^2 \times 2^m \times 2^{-2} - 2^m \times 2^{-2} = 3$

$2^m \left(2^0 - 2^{-2}\right) = 3$

$2^m \times \frac{3}{4} = 3$

$2^m = 4$

$2^m = 2^2$

$m = 2$

Question 9

$x - 10\sqrt{x} + 21 = 0$

$\left(x^{\frac{1}{2}}\right)^2 - 10 \times x^{\frac{1}{2}} + 21 = 0$

Let $a = x^{\frac{1}{2}}$

$a^2 - 10a + 21 = 0$

$(a-7)(a-3) = 0$

$a = 7$ or $a = 3$

$x^{\frac{1}{2}} = 7$ or $x^{\frac{1}{2}} = 3$

$x = 49$ or $x = 9$

Question 10

$5^{2a-b} = \frac{1}{625} \Leftrightarrow 5^{2a-b} = 5^{-4}$

$10^{2b-6a} = 0.01 \Leftrightarrow 10^{2b-6a} = 10^{-2}$

$$\therefore 2a - b = -4 \text{ and } 2b - 6a = -2$$

$4a - 2b = -8 \quad [1]$

$-6a + 2b = -2 \quad [2]$

$$[1] + [2]$$

$$-2a = -10$$

$$a = 5$$

$$20 - 2b = -8$$

$$-2b = -28$$

$$b = 14$$

Exercise 1.03 Logarithmic graphs

Question 1

a $b = 5$

$y = \log_4(x) + 5$

b $b = -3$

$y = \log_{\frac{1}{4}}(x) - 3$

c $b = 2$

$y = \log_7(x) + 2$

d $b = -7$

$y = \log_{0.6}(x) - 7$

Question 2

a $c = 5$

$y = \log_5(x + 5)$

b $c = -3$

$y = \log_{0.4}(x - 3)$

c $c = 2$

$y = \log_{\frac{1}{8}}(x + 2)$

d $c = -7$

$y = \log_3(x - 7)$

Question 3

a $y = \log_2 (x) + 3$

$a = 2$, so $a > 1$

$b = 3$, $\log_2(x)$ is translated 3 units up.

$\text{Zero} = a^{-b}$

$= 2^{-3}$

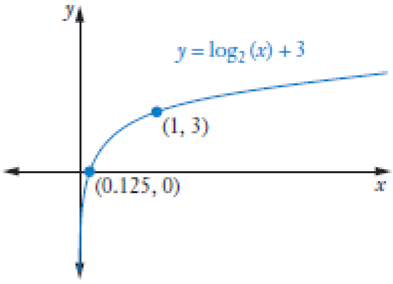
$= \dfrac{1}{8}$

$= 0.125$

The graph passes through (0.125, 0).

The graph passes through (1, b).

(1, b) = (1, 3)



b $y = \log_{0.5}(x) - 2$

$a = 0.5$, so $0 < a < 1$

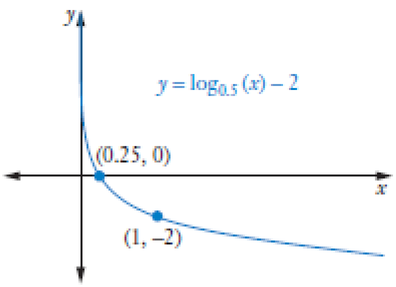
$b = -2$, $\log_{0.5}(x)$ is translated 2 units down.

$$\text{Zero} = a^{-b}$$
$$= 0.5^{-(-2)}$$
$$= 0.25$$

The graph passes through (0.25, 0).

The graph passes through (1, b).

(1, b) = (1, –2)



c $y = \log_5(x) - 1$

$a = 5$, so $a > 1$

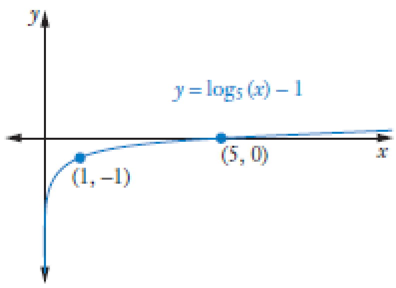
$b = -1$, $\log_5(x)$ is translated 1 units down.

$$\text{Zero} = a^{-b}$$
$$= 5^{-(-1)}$$
$$= 5$$

The graph passes through (5, 0).

The graph passes through (1, *b*).

$(1, b) = (1, -1)$



d $y = \log_{0.8}(x) + 3$

$a = 0.8$, so $0 < a < 1$

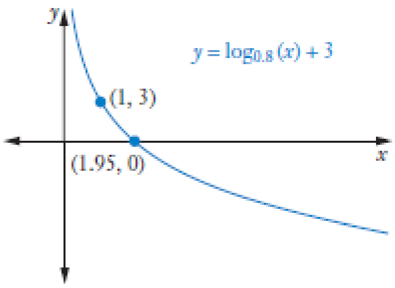
$b = 3$, $\log_{0.8}(x)$ is translated 3 units up.

$$\begin{aligned}\text{Zero} &= a^{-b} \\ &= 0.8^{-3} \\ &= 1.953125... \\ &\approx 1.95\end{aligned}$$

The graph passes through (1.95, 0).

The graph passes through (1, b).

(1, b) = (1, 3)



e $y = \log_3 (x) + 2$

$a = 3$, so $a > 1$

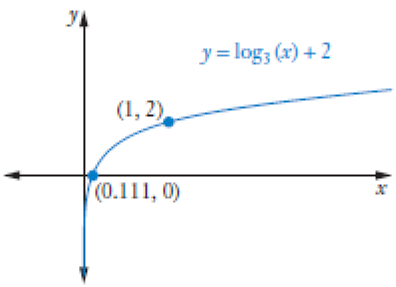
$b = 2$, $\log_3(x)$ is translated 2 units up.

$$\begin{aligned} \text{Zero} &= a^{-b} \\ &= 3^{-2} \\ &= \frac{1}{9} \\ &\approx 0.111 \end{aligned}$$

The graph passes through (0.111, 0).

The graph passes through (1, b).

(1, b) = (1, 2)



f $y = \log_{0.25}(x) - 1$

$a = 0.25$, so $0 < a < 1$

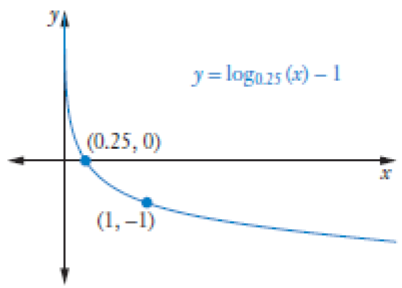
$b = -1$, $\log_{0.25}(x)$ is translated 1 units down.

$$\text{Zero} = a^{-b}$$
$$= 0.25^{-(-1)}$$
$$= 0.25$$

The graph passes through (0.25, 0).

The graph passes through (1, b).

(1, b) = (1, −1)



Question 4

a $y = \log_3 (x + 2)$

$a = 3$, so $a > 1$

$c = 2$, $\log_3(x)$ is translated 2 units to the left.

$$\text{Zero} = 1 - c$$
$$= 1 - 2$$
$$= -1$$

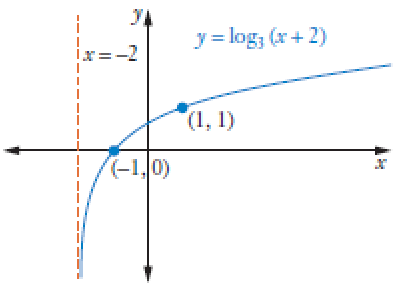
The graph passes through (–1, 0).

$3^1 = 3$, so

$$\log_3(3) = 1$$
$$x + 2 = 3$$
$$x = 1$$

The graph passes through (1, 1).

Vertical asymptote is at $x = -2$.



b $y = \log_{0.5}(x - 2)$

$a = 0.5$, so $0 < a < 1$

$c = -2$, $\log_{0.5}(x)$ is translated 2 units to the right.

$$\begin{aligned} \text{Zero} &= 1 - c \\ &= 1 - (-2) \\ &= 3 \end{aligned}$$

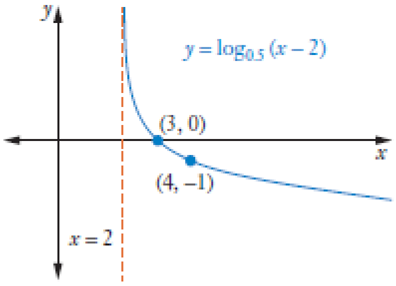
The graph passes through (3, 0).

$(0.5)^{-1} = 2$, so

$$\begin{aligned} \log_{0.5}(2) &= -1 \\ x - 2 &= 2 \\ x &= 4 \end{aligned}$$

The graph passes through (4, −1).

Vertical asymptote is at $x = 2$.



c $y = \log_{0.25}(x + 3)$

$a = 0.25$, so $0 < a < 1$

$c = 3$, $\log_{0.25}(x)$ is translated 3 units to the left.

$$\begin{aligned}\text{Zero} &= 1 - c \\ &= 1 - 3 \\ &= -2\end{aligned}$$

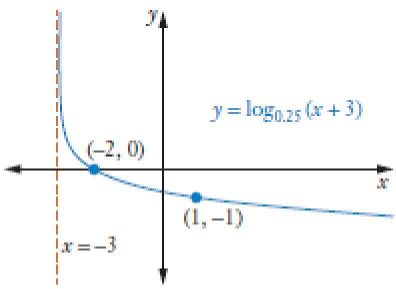
The graph passes through (–2, 0).

$(0.25)^{-1} = 4$, so

$$\begin{aligned}\log_{0.25}(4) &= -1 \\ x + 3 &= 4 \\ x &= 1\end{aligned}$$

The graph passes through (1, –1).

Vertical asymptote is at $x = -3$.



d $y = \log_5 (x + 1)$

$a = 5$, so $a > 1$

$c = 1$, $\log_5(x)$ is translated 1 unit to the left.

$$\begin{aligned}\text{Zero} &= 1 - c \\ &= 1 - 1 \\ &= 0\end{aligned}$$

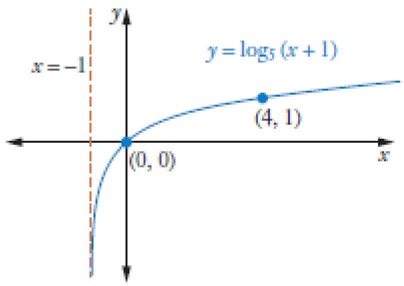
The graph passes through (0, 0).

$5^1 = 5$, so

$$\begin{aligned}\log_5(5) &= 1 \\ x + 1 &= 5 \\ x &= 4\end{aligned}$$

The graph passes through (4, 1).

Vertical asymptote is at $x = -1$.



e $y = \log_4 (x - 4)$

$a = 4$, so $a > 1$

$c = -4$, $\log_4(x)$ is translated 4 units to the right.

$$\begin{aligned}\text{Zero} &= 1 - c \\ &= 1 - (-4) \\ &= 5\end{aligned}$$

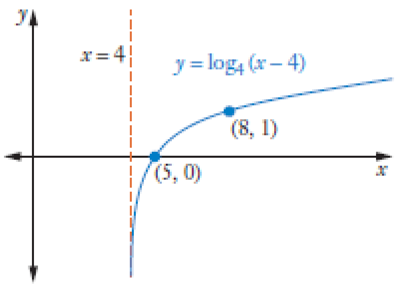
The graph passes through (5, 0).

$4^1 = 4$, so

$$\begin{aligned}\log_4(4) &= 1 \\ x - 4 &= 4 \\ x &= 8\end{aligned}$$

The graph passes through (8, 1).

Vertical asymptote is at $x = 4$.



f $y = \log_{0.6}(x + 2)$

$a = 0.6$, so $0 < a < 1$

$c = 2$, $\log_{0.6}(x)$ is translated 2 units to the left.

$$\begin{aligned} \text{Zero} &= 1 - c \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

The graph passes through (–1, 0).

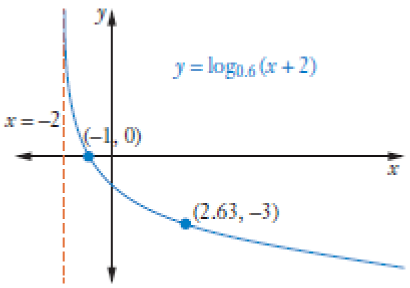
Choose $y = 3$ to find another point.

$$\begin{aligned} (0.6)^{-3} &= 4.6296... \\ &\approx 4.63 \end{aligned}, \text{ so}$$

$$\begin{aligned} \log_{0.6}(4.63) &= -3 \\ x + 2 &= 4.63 \\ x &= 2.63 \end{aligned}$$

The graph passes through (2.63, –3).

Vertical asymptote is at $x = -2$.



Question 5

a $y = \log_2(x + 2) - 1$

$a = 2$, so $a > 1$

$b = -1$, $\log_2(x)$. is translated 1 unit down.

$c = 2$, $\log_2(x)$ is translated 2 units to the left.

x-intercept, let $y = 0$

$0 = \log_2(x+2) - 1$

$1 = \log_2(x+2)$

$2^1 = (x+2)$

$x = 0$

$\therefore$ graph passes through (0, 0).

Let $x = 6$

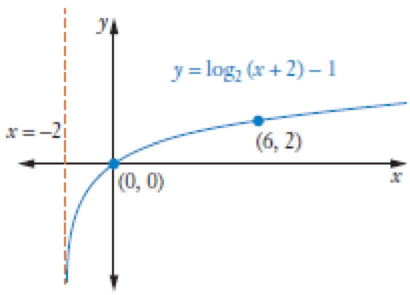
$y = \log_2(8) - 1$

$= 3 - 1$

$= 2$

$\therefore$ graph passes through (6, 2).

Vertical asymptote is at $x = -2$.



b $y = \log_{0.5}(x - 2) + 2$

$a = 0.5$, so $0 < a < 1$

$b = 2$, $\log_{0.5}(x)$ is translated 2 units up.

$c = -2$, $\log_{0.5}(x)$ is translated 2 units to the right.

$\therefore$ graph passes through (3, 2).

x-intercept, let $y = 0$

$$0 = \log_{0.5}(x-2) + 2$$

$$-2 = \log_{0.5}(x-2)$$

$$(0.5)^{-2} = (x-2)$$

$$x - 2 = 4$$

$$x = 6$$

$\therefore$ graph passes through (6, 0).

Let $x = 10$

$$y = \log_{0.5}(8) + 2$$

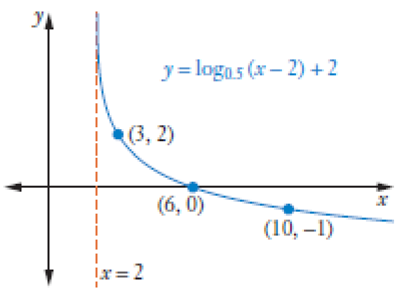
$$= \log_{0.5}(0.5)^{-3} + 2$$

$$= -3 + 2$$

$$= -1$$

$\therefore$ graph passes through (10, –1).

Vertical asymptote is at $x = 2$.



c $y = \log_3 (x - 1) + 2$

$a = 3$, so $a > 1$

$b = 2$, $\log_3(x)$ is translated 2 units up.

$c = -1$, $\log_3(x)$ is translated 1 unit to the right.

x-intercept, let $y = 0$

$$0 = \log_3(x-1) + 2$$

$$-2 = \log_3(x-1)$$

$$3^{-2} = (x-1)$$

$$x - 1 = \frac{1}{9}$$

$$x = \frac{10}{9}$$

$$x \approx 1.11$$

$\therefore$ graph passes through (1.11, 0).

Let $x = 10$

$$y = \log_3(9) + 2$$

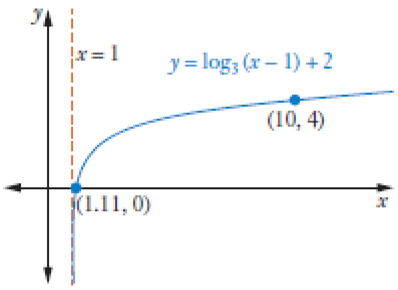
$$= \log_3 3^2 + 2$$

$$= 2 + 2$$

$$= 4$$

$\therefore$ graph passes through (10, 4).

Vertical asymptote is at $x = 1$.



d $y = \log_5 (x + 2) + 1$

$a = 5$, so $a > 1$

$b = 1$, $\log_5(x)$ is translated 1 unit up.

$c = 2$, $\log_5(x)$ is translated 2 units to the left.

x-intercept, let $y = 0$

$$0 = \log_5(x+2)+1$$

$$-1 = \log_5(x+2)$$

$$5^{-1} = (x+2)$$

$$x+2 = \frac{1}{5}$$

$$x = -\frac{9}{5}$$

$$x = -1.8$$

$\therefore$ graph passes through (–1.8, 0).

Let $x = 3$

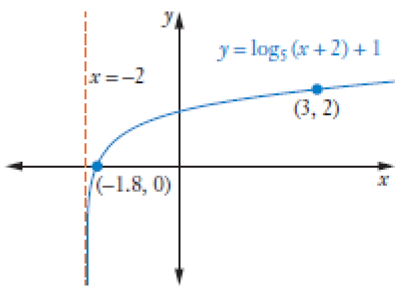
$$y = \log_5(5)+1$$

$$= 1+1$$

$$= 2$$

$\therefore$ graph passes through (3, 2).

Vertical asymptote is at $x = -2$.



Question 6

$y = \log_2 (8x)$

$y = \log_2(8) + \log_2(x)$

$y = \log_2(2^3) + \log_2(x)$

$y = 3\log_2(2) + \log_2(x)$

$y = 3 + \log_2(x)$

$\therefore$ A vertical translation of 3 units up maps $y = \log_2(x)$ to $y = \log_2 (8x)$.

Question 7

$y = \log_3 (x)$

$$\log_3(x) = \frac{\log(x)}{\log(3)}$$

$$\log_9(x) = \frac{\log(x)}{\log(9)}$$

$$= \frac{\log(x)}{\log(3^2)}$$

$$= \frac{\log(x)}{2\log(3)}$$

$$\frac{\log(x)}{\log(3)} = 2\log_9(x)$$

$\therefore \log_3(x) = 2\log_9(x)$

$\therefore$ A vertical dilation with dilation factor of 2.

Exercise 1.04 Logarithmic equations

Question 1

a $\log_5 (7x + 3) = \log_5 (5x + 9)$

$(7x + 3) = (5x + 9)$

$2x = 6$

$x = 3$

Check: Let $x = 3$

$$\log_5(21+3) = \log_5(15+9)$$
$$\log_5(24) = \log_5(24)$$

$\therefore x = 3$ is a solution.

b $\log_8 (x) + \log_8 (x + 6) = \log_8 (5x + 12)$

$$\log_8(x(x+6)) = \log_8(5x+12)$$
$$x^2 + 6x = 5x + 12$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x = -4 \text{ or } x = 3$$

Check: Let $x = -4$

$$\log_8(-4) + \log_8(-4+6) = \log_8(-20+12)$$

$x = -4$ gives a negative number in the logarithms, so $x = -4$ is not a solution.

Check: Let $x = 3$

$$\log_8(3) + \log_8(3+6) = \log_8(15+12)$$
$$\log_8(3\times 9) = \log_8(27)$$
$$\log_8(27) = \log_8(27)$$

$\therefore x = 3$ is a solution.

c log (x – 2) – log (2x – 3) = log (2)

$$\log\left(\frac{(x-2)}{(2x-3)}\right)=\log(2)$$

$$(x-2)=2(2x-3)$$

$$x-2=4x-6$$

$$3x=4$$

$$x=\frac{4}{3}$$

Check: Let $x = \frac{4}{3}$

$$\log\left(\frac{\left(\frac{4}{3}-2\right)}{\left(2\times\frac{4}{3}-3\right)}\right)=\log(2)$$

$$\log\left(\frac{-\frac{2}{3}}{\frac{-1}{3}}\right)=\log(2)$$

$$\log(2)=\log(2)$$

$\therefore x = \frac{4}{3}$ is a solution.

d $\log_3 [(x-2)(x+3)] = \log_3 (14)$

$$(x-2)(x+3)=14$$

$$x^2+x-6=14$$

$$x^2+x-20=0$$

$$(x+5)(x-4)=0$$

$$x=-5 \text{ or } x=4$$

Check: Let $x = -5$

$$\log_3(-7\times-2)=\log_3(14)$$

$$\log_3(14)=\log_3(14)$$

Check: Let $x = 4$

$$\log_3(2\times7)=\log_3(14)$$

$$\log_3(14)=\log_3(14)$$

$\therefore x = -5$ and $x = 4$ are solutions.

e $\log_6 (2x + 1) = \log_6 (x + 2) - \log_6 (3)$

$$\log_6 (2x+1) = \log_6 \left(\frac{x+2}{3}\right)$$

$$2x+1 = \frac{x+2}{3}$$

$$6x+3 = x+2$$

$$5x = -1$$

$$x = -\frac{1}{5}$$

Check: Let $x = -\frac{1}{5}$

$$\log_6 \left(-\frac{2}{5}+1\right) = \log_6 \left(\frac{-\frac{1}{5}+2}{3}\right)$$

$$\log_6 \left(\frac{3}{5}\right) = \log_6 \left(\frac{9}{5}\times\frac{1}{3}\right)$$

$$\log_6 \left(\frac{3}{5}\right) = \log_6 \left(\frac{3}{5}\right)$$

$\therefore x = -\frac{1}{5}$ is a solution.

f $\log_7 (x + 4) + \log_7 (x - 2) = \log_7 (4x)$

$$\log_7\left((x+4)(x-2)\right) = \log_7(4x)$$

$$x^2 + 2x - 8 = 4x$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x = 4 \text{ or } x = 2$$

Check: Let $x = 4$

$$\log_7(8) + \log_7(2) = \log_7(4\times 4)$$

$$\log_7(8\times 2) = \log_7(16)$$

$$\log_7(16) = \log_7(16)$$

$\therefore x = 4$ is a solution

Check: Let $x = 2$

$$\log_7(6) + \log_7(0) = \log_7(4\times 2)$$

$$\log_7(6\times 0) = \log_7(8)$$

$$\log_7(0) \neq \log_7(8)$$

$\log_7(0)$ is undefined, $\therefore x = 2$ is not a solution.

g log (3*x* + 5) = log (7*x* – 12)

$$3x+5=7x-12$$

$$-4x=-17$$

$$x=\frac{17}{4}$$

Check: Let $x = \frac{17}{4}$

$$\log\left(3\times\frac{17}{4}+5\right)=\log\left(7\times\frac{17}{4}-12\right)$$

$$\log\left(\frac{71}{4}\right)=\log\left(\frac{71}{4}\right)$$

$\therefore x = \frac{17}{4}$ is a solution.

h $\log_2 (3x + 2) = \log_2 (x - 2)$

$$3x+2=x-2$$

$$2x=-4$$

$$x=-2$$

Check: Let *x* = –2

LHS = $\log_2 (3 \times -2 + 2) = \log_2 (-4)$

RHS = $\log_2 (-2 - 2) = \log_2 (-4)$

So LHS = RHS, but both undefined.

So no solution.

i $\log_6(2x) + \log_6(4) = \log_6(x + 12) - \log_6(2)$

$$\log_6(2x \times 4) = \log_6\left(\frac{x+12}{2}\right)$$

$$8x = \frac{x+12}{2}$$

$$16x = x + 12$$

$$15x = 12$$

$$x = \frac{12}{15} = \frac{4}{5}$$

Check: Let $x = \frac{4}{5}$

$$\log_6\left(2 \times \frac{4}{5} \times 4\right) = \log_6\left(\frac{\frac{4}{5}+12}{2}\right)$$

$$\log_6\left(\frac{32}{5}\right) = \log_6\left(\frac{64}{5} \times \frac{1}{2}\right)$$

$$\log_6\left(\frac{32}{5}\right) = \log_6\left(\frac{32}{5}\right)$$

$\therefore x = \frac{4}{5}$ is a solution.

j $\log_8(2x + 1) + \log_8(2x - 1) = 3\log_8(3)$

$$\log_8(2x+1)(2x-1) = \log_8(3^3)$$

$$\log_8(4x^2-1) = \log_8(27)$$

$$4x^2 - 1 = 27$$

$$4x^2 = 28$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

Check: Let $x = -\sqrt{7}$

$$\log_8(2\times-\sqrt{7}+1) + \log_8(2\times-\sqrt{7}-1) = 3\log_8(3)$$

$$\log_8(-4.29) + \log_8(-6.29) = 3\log_8(3)$$

Log of negative number is undefined, $\therefore x = -\sqrt{7}$ is not a solution.

Check: Let $x = \sqrt{7}$

$$\log_8(2\times\sqrt{7}+1) + \log_8(2\times\sqrt{7}-1) = 3\log_8(3)$$

$$\log_8\left[(2\times\sqrt{7}+1)(2\times\sqrt{7}-1)\right] = \log_8(3^3)$$

$$\log_8(4\times7-1) = \log_8(27)$$

$$\log_8(27) = \log_8(27)$$

$\therefore x = \sqrt{7}$ is a solution.

Question 2

a $\log_2 (5x + 7) = 5$

$$\log_2\left(5x+7\right)=5\log_2\left(2\right)$$

$$\log_2\left(5x+7\right)=\log_2\left(2^5\right)$$

$$5x+7=32$$

$$5x=25$$

$$x=5$$

Check: Let $x = 5$

$$\log_2\left(5\times5+7\right)=5$$

$$\log_2\left(32\right)=5$$

$$\log_2\left(2^5\right)=5$$

$$5\log_2\left(2\right)=5$$

$$5=5$$

$\therefore x = 5$ is a solution.

b log (4*x* – 1) = 3

$\log(4x-1)=3\log(10)$

$\log(4x-1)=\log(10^3)$

$4x-1=1000$

$4x=1001$

$x=250.25$

Check: Let *x* = 250.25

$\log(4\times 250.25-1)=3$

$\log(1000)=3$

$\log(10^3)=5$

$3\log(10)=3$

$3=3$

$\therefore x$ = 250.25 is a solution.

c $\log_3 (5x - 11) = 2$

$$\log_3(5x-11) = 2\log_3(3)$$

$$\log_3(5x-11) = \log_3(3^2)$$

$$5x - 11 = 9$$

$$5x = 20$$

$$x = 4$$

Check: Let $x = 4$

$$\log_3(5\times 4 - 11) = 2$$

$$\log_3(9) = 2$$

$$\log_3(3^2) = 2$$

$$2\log_3(3) = 2$$

$$2 = 2$$

$\therefore x = 4$ is a solution.

d $\log_5 (4x + 11) = 2$

$$\log_5\left(4x+11\right)=2\log_5\left(5\right)$$

$$\log_5\left(4x+11\right)=\log_5\left(5^2\right)$$

$$4x+11=25$$

$$4x=14$$

$$x=\frac{7}{2}$$

Check: Let $x = \frac{7}{2}$

$$\log_5\left(4\times\frac{7}{2}+11\right)=2$$

$$\log_5\left(25\right)=2$$

$$\log_5\left(5^2\right)=2$$

$$2\log_5\left(5\right)=2$$

$$2=2$$

$\therefore x = \frac{7}{2}$ is a solution.

e $\log_3 (9x + 2) = 4$

$$\log_3(9x+2) = 4\log_3(3)$$

$$\log_3(9x+2) = \log_3(3^4)$$

$$9x+2 = 81$$

$$9x = 79$$

$$x = \frac{79}{9} = 8\frac{7}{9}$$

Check: Let $x = \frac{79}{9}$.

$$\log_3\left(9\times\frac{79}{9}+2\right) = 4$$

$$\log_3(79+2) = 4$$

$$\log_3(81) = 4$$

$$\log_3(3^4) = 4$$

$$4 = 4$$

$\therefore$ $x = \frac{79}{9}$ is a solution.

f $\log_3(4x-9) = 3$

$$\log_3(4x-9) = 3\log_3(3)$$

$$\log_3(4x-9) = \log_3(3^3)$$

$$4x-9 = 27$$

$$4x = 36$$

$$x = 9$$

Check: Let $x = 9$

$$\log_3(4\times9-9) = 3$$

$$\log_3(27) = 3$$

$$\log_3(3^3) = 3$$

$$3\log_3(3) = 3$$

$$3 = 3$$

$\therefore x = 9$ is a solution.

g $\log_3(3x + 11) = 4$

$$\log_3(3x+11) = 4\log_3(3)$$

$$\log_3(3x+11) = \log_3(3^4)$$

$$3x+11 = 81$$

$$3x = 70$$

$$x = \frac{70}{3} = 23\frac{1}{3}$$

Check: Let $x = \frac{70}{3}$

$$\log_3\left(3\times\frac{70}{3}+11\right) = 4$$

$$\log_3(81) = 4$$

$$\log_3(3^4) = 4$$

$$4\log_3(3) = 4$$

$$4 = 4$$

$\therefore x = \frac{70}{3}$ is a solution.

h $\log_7 (6x - 5) = 2$

$$\log_7\left(6x-5\right)=2\log_7\left(7\right)$$

$$\log_7\left(6x-5\right)=\log_7\left(7^2\right)$$

$$6x-5=49$$

$$6x=54$$

$$x=9$$

Check: Let $x = 9$

$$\log_7\left(6\times9-5\right)=2$$

$$\log_7\left(49\right)=2$$

$$\log_7\left(7^2\right)=2$$

$$2\log_7\left(7\right)=2$$

$$2=2$$

$\therefore x = 9$ is a solution.

i $\log_3 (x^2 - 6x) = 3$

$$\log_3\left(x^2 - 6x\right) = 3\log_3\left(3\right)$$

$$\log_3\left(x^2 - 6x\right) = \log_3\left(3^3\right)$$

$$x^2 - 6x = 27$$

$$x^2 - 6x - 27 = 0$$

$$\left(x-9\right)\left(x+3\right) = 0$$

$$x = 9 \text{ or } x = -3$$

Check: Let $x = 9$

$$\log_3\left(9^2 - 6\times 9\right) = 3$$

$$\log_3\left(27\right) = 3$$

$$\log_3\left(3^3\right) = 3$$

$$3\log_3\left(3\right) = 3$$

$$3 = 3$$

$\therefore x = 9$ is a solution.

Check: Let $x = -3$

$$\log_3\left(\left(-3\right)^2 - 6\times\left(-3\right)\right) = 3$$

$$\log_3\left(27\right) = 3$$

$$\log_3\left(3^3\right) = 3$$

$$3\log_3\left(3\right) = 3$$

$$3 = 3$$

$\therefore x = -3$ is a solution.

Question 3

a $\log_2 (x + 5) - \log_2 (2x - 1) = 5$

$$\log_2\left(\frac{x+5}{2x-1}\right) = 5\log_2 2$$

$$\log_2\left(\frac{x+5}{2x-1}\right) = \log_2\left(2^5\right)$$

$$\frac{x+5}{2x-1} = 32$$

$$x+5 = 32(2x-1)$$

$$x+5 = 64x-32$$

$$63x = 37$$

$$x = \frac{37}{63}$$

Check: Let $x = \frac{37}{63}$

$$\log_2\left(\frac{\frac{37}{63}+5}{2\times\frac{37}{63}-1}\right) = 5$$

$$\log_2(32) = 5$$

$$\log_2\left(2^5\right) = 5$$

$$5\log_2(2) = 5$$

$$5 = 5$$

$\therefore x = \frac{37}{63}$ is a solution.

b $2\log_3(x) - 1 = \log_3(2x - 3)$

$$2\log_3(x) - \log_3 3 = \log_3(2x-3)$$

$$\log_3(x^2) - \log_3 3 = \log_3(2x-3)$$

$$\log_3\left(\frac{x^2}{3}\right) = \log_3(2x-3)$$

$$\frac{x^2}{3} = 2x-3$$

$$x^2 = 6x-9$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3$$

Check: Let $x = 3$

$$2\log_3(3) - 1 = \log_3(6-3)$$

$$2\log_3(3) - 1 = \log_3(3)$$

$$2\times1 - 1 = 1$$

$$1 = 1$$

$\therefore$ $x = 3$ is a solution.

c $\log_4(4x-2)-\log_4(3x+1)=5$

$$\log_4\left(\frac{4x-2}{3x+1}\right)=5\log_4(4)$$

$$\log_4\left(\frac{4x-2}{3x+1}\right)=\log_4\left(4^5\right)$$

$$\frac{4x-2}{3x+1}=1024$$

$$4x-2=1024(3x+1)$$

$$-3068x=1026$$

$$x=-\frac{1026}{3068}=-\frac{513}{1534}$$

Check: Let $x = -\frac{513}{1534}$

$$\log_4\left(4\times-\frac{513}{1534}-2\right)-\log_4\left(3\times-\frac{513}{1534}+1\right)=5$$

$$\log_4\left(-\frac{5120}{1534}\right)-\log_4\left(-\frac{5}{1534}\right)=5$$

Log of a negative number is undefined.

$\therefore x = -\frac{513}{1534}$ is not a solution.

$\therefore$ No solution possible.

d $\log_2 (x + 1) - \log_2 (x - 4) = 3$

$$\log_2\left(\frac{(x+1)}{(x-4)}\right) = 3\log_2 2$$

$$\log_2\left(\frac{(x+1)}{(x-4)}\right) = \log_2\left(2^3\right)$$

$$\frac{(x+1)}{(x-4)} = 8$$

$$x+1 = 8(x-4)$$

$$x+1 = 8x-32$$

$$7x = 33$$

$$x = \frac{33}{7}$$

Check: Let $x = \frac{33}{7}$

$$\log_2\left(\frac{33}{7}+1\right) - \log_2\left(\frac{33}{7}-4\right) = 3$$

$$\log_2\left(\frac{40}{7}\right) - \log_2\left(\frac{5}{7}\right) = 3$$

$$\log_2(8) = 3$$

$$\log_2\left(2^3\right) = 3$$

$$3\log_2(2) = 3$$

$$3 = 3$$

$\therefore x = \frac{33}{7}$ is a solution.

e $\log_2(x) + \log_2(2 - 2x) = -1$

$$\log_2(x) + \log_2(2-2x) = -\log_2(2)$$

$$\log_2(x(2-2x)) = \log_2(2^{-1})$$

$$2x - 2x^2 = \frac{1}{2}$$

$$4x^2 - 4x + 1 = 0$$

$$(2x-1)^2 = 0$$

$$x = \frac{1}{2}$$

Check: Let $x = \frac{1}{2}$

$$\log_2\left(\frac{1}{2}\right) + \log_2\left(2 - 2\times\frac{1}{2}\right) = -1$$

$$\log_2(2^{-1}) + \log_2(1) = -1$$

$$-1 + 0 = -1$$

$$-1 = -1$$

$\therefore x = \frac{1}{2}$ is a solution.

f $\log_3(x) + \log_3(2x + 1) = 1$

$$\log_3(x)+\log_3(2x+1)=\log_3(3)$$

$$\log_3(x(2x+1))=\log_3(3)$$

$$2x^2+x=3$$

$$2x^2+x-3=0$$

$$(2x+3)(x-1)=0$$

$$x=-\frac{3}{2}\text{ or } x=1$$

Check: Let $x = -\frac{3}{2}$

$$\log_3\left(-\frac{3}{2}\right)+\log_3\left(2\times-\frac{3}{2}+1\right)=1$$

Log of a negative number is undefined.

$\therefore x = -\frac{3}{2}$ is not a solution.

Check: Let $x = 1$

$$\log_3(1)+\log_3(2\times1+1)=1$$

$$0+1=1$$

$$1=1$$

$\therefore x = 1$ is a solution.

g $\log_6(x) + \log_6(x+5) = 2$

$$\log_6\left(x(x+5)\right) = 2\log_6(6)$$

$$\log_6\left(x^2+5x\right) = \log_6\left(6^2\right)$$

$$x^2+5x = 36$$

$$x^2+5x-36 = 0$$

$$(x+9)(x-4) = 0$$

$$x = -9 \text{ or } x = 4$$

Check: Let $x = -9$

$$\log_6(-9) + \log_6(-9+5) = 2$$

Log of a negative number is undefined.

$\therefore x = -9$ is not a solution.

Check: Let $x = 4$

$$\log_6(4) + \log_6(4+5) = 2$$

$$\log_6(36) = 2$$

$$\log_6\left(6^2\right) = 2$$

$$2\log_6(6) = 2$$

$$2 = 2$$

$\therefore x = 4$ is a solution.

h $\log_2(2x + 12) - \log_2(3x) = 2$

$$\log_2\left(\frac{2x+12}{3x}\right) = 2\log_2(2)$$

$$\log_2(\quad) = \log_2\left(2^2\right)$$

$$\frac{2x+12}{3x} = 4$$

$$2x + 12 = 12x$$

$$10x = 12$$

$$x = \frac{6}{5}$$

Check: Let $x = \dfrac{6}{5}$

$$\log_2\left(2\times\frac{6}{5}+12\right) - \log_2\left(3\times\frac{6}{5}\right) = 2$$

$$\log_2\left(\frac{72}{5}\right) - \log_2\left(\frac{18}{5}\right) = 2$$

$$\log_2\left(\frac{72}{5}\div\frac{18}{5}\right) = 2$$

$$\log_2(4) = 2$$

$$\log_2\left(2^2\right) = 2$$

$$2\log_2(2) = 2$$

$$2 = 2$$

$\therefore x = \dfrac{6}{5}$ is a solution.

i $\log_4(2x-3) - \log_4(x+2) = 1$

$$\log_4\left(\frac{2x-3}{x+2}\right)=1$$

$$\left(\frac{2x-3}{x+2}\right)=4^1=4$$

$$2x-3=4(x+2)$$

$$-2x=11$$

$$x=-\frac{11}{2}$$

Check: Let $x = -\frac{11}{2}$

$$\log_4\left(2\times-\frac{11}{2}-3\right)-\log_4\left(-\frac{11}{2}+2\right)=1$$

$$\log_4(-8)-\log_4\left(-\frac{7}{2}\right)=1$$

Log of a negative number is undefined.

$\therefore x = -\frac{11}{2}$ is not a solution.

$\therefore$ No solution possible.

j $\log_3(x) - \log_3(x-1) = 2$

$$\log_3\left(\frac{x}{x-1}\right) = 2\log_3(3)$$

$$\log_3\left(\frac{x}{x-1}\right) = \log_3\left(3^2\right)$$

$$\frac{x}{x-1} = 9$$

$$x = 9(x-1)$$

$$-8x = -9$$

$$x = \frac{9}{8}$$

Check: Let $x = \frac{9}{8}$

$$\log_3\left(\frac{9}{8}\right) - \log_3\left(\frac{9}{8} - 1\right) = 2$$

$$\log_3\left(\frac{9}{8}\right) - \log_3\left(\frac{1}{8}\right) = 2$$

$$\log_3\left(\frac{9}{8} \div \frac{1}{8}\right) = 2$$

$$\log_3(9) = 2$$

$$\log_3\left(3^2\right) = 2$$

$$2\log_3(3) = 2$$

$$2 = 2$$

$\therefore x = \frac{9}{8}$ is a solution.

Question 4

a $\log_6(x) + \log_6(x-9) = 2$

$$\log_6\left(x\left(x-9\right)\right) = 2\log_6\left(6\right)$$

$$\log_6\left(x^2-9x\right) = \log_6\left(6^2\right)$$

$$x^2-9x = 36$$

$$x^2-9x-36 = 0$$

$$\left(x-12\right)\left(x+3\right) = 0$$

$$x = -3 \text{ or } x = 12$$

Check: Let $x = -3$

$$\log_6\left(-3\right)+\log_6\left(-3-9\right) = 2$$

Log of a negative number is undefined.

$\therefore x = -3$ is not a solution.

Check: Let $x = 12$

$$\log_6\left(12\right)+\log_6\left(12-9\right) = 2$$

$$\log_6\left(36\right) = 2$$

$$\log_6\left(6^2\right) = 2$$

$$2\log_6\left(6\right) = 2$$

$$2 = 2$$

$\therefore x = 12$ is a solution.

b $\log_4 (x) + \log_4 (x - 12) = 3$

$$\log_4\left(x\left(x-12\right)\right)=3\log_4\left(4\right)$$

$$\log_4\left(x^2-12x\right)=\log_4\left(4^3\right)$$

$$x^2-12x=64$$

$$x^2-12x-64=0$$

$$\left(x-16\right)\left(x+4\right)=0$$

$$x=-4=16$$

Check: Let $x = -4$

$$\log_4\left(-4\right)+\log_4\left(-4-12\right)=3$$

Log of a negative number is undefined.

$\therefore x = -4$ is not a solution.

Check: Let $x = 16$

$$\log_4\left(16\right)+\log_4\left(16-12\right)=3$$

$$\log_4\left(4^2\right)+\log_4\left(4\right)=3$$

$$2\log_4\left(4\right)+\log_4\left(4\right)=3$$

$$2+1=3$$

$$3=3$$

$\therefore x = 16$ is a solution.

c $\log_6(x) + \log_6(x-1) = 1$

$$\log_6\left(x(x-1)\right) = \log_6(6)$$

$$\log_6\left(x^2 - x\right) = \log_6(6)$$

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3) = 0$$

$$x = -2 \text{ or } x = 3$$

Check: Let $x = -2$

$$\log_6(-2) + \log_6(-2-1) = 1$$

Log of a negative number is undefined.

$\therefore x = -2$ is not a solution.

Check: Let $x = 3$

$$\log_6(3) + \log_6(3-1) = 1$$

$$\log_6(6) = 1$$

$$1 = 1$$

$\therefore x = 3$ is a solution.

d $\log_6 (x^2 - 2x) = \log_6 (5x - 12)$

$$x^2 - 2x = 5x - 12$$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$x = 3 \text{ or } x = 4$$

Check: Let $x = 3$

$$\log_6 (3^2 - 2\times 3) = \log_6 (5\times 3 - 12)$$

$$\log_6 (3) = \log_6 (3)$$

$\therefore x = 3$ is a solution.

Check: Let $x = 4$

$$\log_6 (4^2 - 2\times 4) = \log_6 (5\times 4 - 12)$$

$$\log_6 (8) = \log_6 (8)$$

$\therefore x = 4$ is a solution.

e $\log_7(6x) - \log_7(4-x) = \log_7(3)$

$$\log_7\left(\frac{6x}{4-x}\right) = \log_7(3)$$

$$\frac{6x}{4-x} = 3$$

$$6x = 3(4-x)$$

$$6x = 12 - 3x$$

$$9x = 12$$

$$x = \frac{4}{3}$$

Check: Let $x = \frac{4}{3}$

$$\log_7\left(6\times\frac{4}{3}\right) - \log_7\left(4-\frac{4}{3}\right) = \log_7(3)$$

$$\log_7(8) - \log_7\left(\frac{8}{3}\right) = \log_7(3)$$

$$\log_7\left(8\div\frac{8}{3}\right) = \log_7(3)$$

$$\log_7(3) = \log_7(3)$$

$\therefore x = \frac{4}{3}$ is a solution

f $\log(x) + \log(x+3) = \log(5x)$

$$\log\left[x(x+3)\right] = \log(5x)$$

$$x^2 + 3x = 5x$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, 2$$

But log (0) is not defined.

$\therefore\ x = 2.$

g $\log_5 (x - 1) + \log_5 (x + 3) = \log_5 (x^2 - 3x + 12)$

$$\log_5\left[(x-1)(x+3)\right] = \log_5\left(x^2 - 3x + 12\right)$$

$$\log_5\left(x^2 + 2x - 3\right) = \log_5\left(x^2 - 3x + 12\right)$$

$$x^2 + 2x - 3 = x^2 - 3x + 12$$

$$5x = 15$$

$$x = 3$$

Check: Let $x = 3$

$$\log_5(3-1) + \log_5(3+3) = \log_5\left(3^2 - 3\times 3 + 12\right)$$

$$\log_5(2) + \log_5(6) = \log_5(12)$$

$$\log_5(2\times 6) = \log_5(12)$$

$$\log_5(12) = \log_5(12)$$

$\therefore x = 3$ is a solution

h $[\log_3(x)]^2 = 3 - 2\log_3(x)$

Let $a = \log_3(x)$

$$a^2 = 3 - 2a$$

$$a^2 + 2a - 3 = 0$$

$$(a+3)(a-1) = 0$$

$$a = -3 \text{ or } a = 1$$

$\log_3(x) = -3$ $\quad\quad$ $\log_3(x) = 1$

$x = 3^{-3}$ $\quad\quad$ $x = 3^1$

$x = \frac{1}{27}$ $\quad\quad$ $x = 3$

Check $x = \frac{1}{27}$

$$\left[\log_3\left(\frac{1}{27}\right)\right]^2 = 3 - 2\log_3\left(\frac{1}{27}\right)$$

$$\left[\log_3\left(3^{-3}\right)\right]^2 = 3 - 2\log_3\left(3^{-3}\right)$$

$$\left[-3\log_3(3)\right]^2 = 3 - 2 \times -3\log_3(3)$$

$$9 = 3 + 6$$

$$9 = 9$$

$\therefore x = \frac{1}{27}$ is a solution.

Check: Let $x = 3$

$$\left[\log_3(3)\right]^2 = 3 - 2\log_3(3)$$

$$1 = 3 - 2$$

$$1 = 1$$

$\therefore x = 3$ is a solution.

i $\log_5(2x) + \log_5(x+2) = \log_5(6)$

$$\log_5\left(2x(x+2)\right) = \log_5(6)$$

$$\log_5\left(2x^2 + 2x\right) = \log_5(6)$$

$$2x^2 + 2x = 6$$

$$x^2 + x - 6 = 0$$

$$(x+5)(x-1) = 0$$

$$x = -5 \text{ or } x = 1$$

Check: Let $x = -5$

$$\log_5(2\times -5) + \log_5(-5+2) = \log_5(6)$$

$$\log_5(-10) + \log_5(-3) \neq \log_5(6)$$

$\therefore x = -5$ is not a solution.

Check: Let $x = 1$

$$\log_5(2\times 1) + \log_5(1+2) = \log_5(6)$$

$$\log_5(2) + \log_5(3) = \log_5(6)$$

$$\log_5(6) = \log_5(6)$$

$\therefore x = 1$ is a solution.

j $\log_8 (2) + \log_8 (4x^2) = 2$

$$\log_8\left(2\times 4x^2\right) = 2\log_8\left(8\right)$$

$$\log_8\left(8x^2\right) = \log_8\left(8^2\right)$$

$$8x^2 = 64$$

$$x^2 - 8 = 0$$

$$\left(x-2\sqrt{2}\right)\left(x+2\sqrt{2}\right) = 0$$

$$x = \pm 2\sqrt{2}$$

Check: Let $x = -2\sqrt{2}$

$$\log_8\left(2\right)+\log_8\left(4\times\left(-2\sqrt{2}\right)^2\right) = 2$$

$$\log_8\left(2\right)+\log_8\left(32\right) = 2$$

$$\log_8\left(64\right) = 2$$

$$\log_8\left(8^2\right) = 2$$

$$2\log_8\left(8\right) = 2$$

$$2 = 2$$

$\therefore x = -2\sqrt{2}$ is a solution.

Check: Let $x = 2\sqrt{2}$

$$\log_8\left(2\right)+\log_8\left(4\times\left(2\sqrt{2}\right)^2\right) = 2$$

$$\log_8\left(2\right)+\log_8\left(32\right) = 2$$

$$\log_8\left(64\right) = 2$$

$$\log_8\left(8^2\right) = 2$$

$$2\log_8\left(8\right) = 2$$

$$2 = 2$$

$\therefore x = 2\sqrt{2}$ is a solution.

Question 5

$2^{\log_2(x-5)} + 2(x-5) - 12 = 0$

$(x-5)+2(x-5)-12=0$

$3x-27=0$

$3x=27$

$x=9$

Check: Let $x = 9$

$$\begin{aligned} 2^{\log_2(9-5)}+2(9-5)-12&=0 \\ 2^{\log_2(4)}+2\times4-12&=0 \\ 2^{\log_2\left(2^2\right)}-4&=0 \\ 2\times2^{\log_2(2)}-4&=0 \\ 2\times2^1-4&=0 \\ 0&=0 \end{aligned}$$

Question 6

$\log_{10}(5^x + x - 31) = x[1 - \log_{10}(2)]$

$\log_{10}\left(5^x+x-31\right)=x\left[\log_{10}(10)-\log_{10}(2)\right]$

$\log_{10}\left(5^x+x-31\right)=x\log_{10}(5)$

$\log_{10}\left(5^x+x-31\right)=\log_{10}\left(5^x\right)$

$$\begin{aligned} 5^x+x-31&=5^x \\ x&=31 \end{aligned}$$

Exercise 1.05 Applications of logarithmic functions

Question 1

$$M(x) = \log\left(\frac{x}{x_0}\right)$$

$$\begin{aligned} M(x) &= \log\left(\frac{758x_0}{x_0}\right) \\ &= \log(758) \\ &= 2.87966... \\ &\approx 2.9 \end{aligned}$$

Question 2

$L = 10\log\left(\frac{I}{I_0}\right)$

Drill:

$$50 = 10\log\left(\frac{I}{10^{-12}}\right)$$

$$5 = \log\left(10^{12} I\right)$$

$$10^5 = 10^{12} I$$

$$I = 10^{-7}\ \text{W/m}^2$$

Compressor:

$$62 = 10\log\left(\frac{I}{10^{-12}}\right)$$

$$6.2 = \log\left(10^{12} I\right)$$

$$10^{6.2} = 10^{12} I$$

$$I = 10^{-5.8}\ \text{W/m}^2$$

$$\frac{10^{-5.8}}{10^{-7}} = 10^{1.2}$$

$$= 15.8489...$$

$$\approx 15.85$$

$\therefore$ Compressor noise is $10^{1.2}$ or approximately 15.85 W/m^2 more intense than drill noise.

Question 3

$$FV = PV\left(1+\frac{r}{n}\right)^{nt}$$

$$FV = 2PV$$

$$2PV = PV\left(1+\frac{0.1}{12}\right)^{12t}$$

$$2 = \left(1+\frac{0.1}{12}\right)^{12t}$$

$$\log(2) = \log(1.0083...)^{12t}$$

$$\log(2) = 12t\log(1.0083...)$$

$$12t = \frac{\log(1.0083...)}{\log(2)}$$

$$12t = 83.5237...$$

$$t = 6.9603...$$

$$t \approx 7 \text{ years}$$

Question 4

$M(x) = \log\left(\frac{x}{x_0}\right)$

$8.3 = \log\left(\frac{x}{x_0}\right)$

$10^{8.3} = \frac{x}{x_0}$

$x = 10^{8.3} x_0$

$$
\begin{aligned}
M(x) &= \log\left(\frac{4\times 10^{8.3} x_0}{x_0}\right) \\
&= \log\left(4\times 10^{8.3}\right) \\
&= 8.90205... \\
&\approx 8.9
\end{aligned}
$$

Chilean earthquake measured approximately 8.9 on the Richter scale.

Question 5

$M(x) = \log\left(\frac{x}{x_0}\right)$

Ecuador:

$8.3 = \log\left(\frac{x_1}{x_0}\right)$

$10^{8.3} = \frac{x_1}{x_0}$

$x_1 = 10^{8.3}x_0$

New Zealand:

$7.1 = \log\left(\frac{x_2}{x_0}\right)$

$10^{7.1} = \frac{x_2}{x_0}$

$x_2 = 10^{7.1}x_0$

$\frac{x_1}{x_2} = \frac{10^{8.3}x_0}{10^{7.1}x_0}$

$= 10^{1.2}$

$= 15.8489...$

≈ 15.85

The earthquake in Ecuador is approximately 16 times greater.

Question 6

Amp 1: 25 W/m^2

$$L = 10\log\left(\frac{25}{10^{-12}}\right)$$

$$= 133.9794...$$

$$\approx 133.98$$

Amp 2: 500 W/m^2

$$L = 10\log\left(\frac{500}{10^{-12}}\right)$$

$$= 147.7815...$$

$$\approx 147.78$$

Difference = 147.78 − 133.98 = 13.8 dB

Question 7

L = 160 dB $\quad$ I = ?

$$160 = 10\log\left(\frac{I}{10^{-12}}\right)$$

$$160 = 10\log\left(10^{12} I\right)$$

$$16 = \log\left(10^{12} I\right)$$

$$10^{16} = 10^{12} I$$

$$I = 10^{4}\ \text{W/m}^2$$

Question 8

a $$¢ = \frac{1200}{\log(2)}\log\left(\frac{f_2}{f_1}\right)$$

$f_1 = 27.5, \quad f_2 = 29.135$

$$¢ = \frac{1200}{\log(2)}\log\left(\frac{29.135}{27.5}\right)$$

$$= 99.986...$$

$$\approx 100$$

$\therefore$ There are 100 ¢ between A and B flat.

b $$¢ = \frac{1200}{\log(2)}\log\left(\frac{f_2}{f_1}\right)$$

$f_1 = 261.63, \quad f_2 = 440$

$$¢ = \frac{1200}{\log(2)}\log\left(\frac{440}{261.63}\right)$$

$$= 899.97...$$

$$\approx 900$$

$\therefore$ There are 900 ¢ between middle C and A4.

c $$¢ = \frac{1200}{\log(2)}\log\left(\frac{f_2}{f_1}\right)$$

$f_1 = 261.63, \quad f_2 = 587.33$

$$¢ = \frac{1200}{\log(2)}\log\left(\frac{587.33}{261.63}\right)$$

$$= 1399.972...$$

$$\approx 1400$$

$\therefore$ There are 1400 ¢ between middle C and C5.

Question 9

$pH = -\log[H^+]$

$pH = 1.8$, $[H^+] = ?$

$$1.8 = -\log\left[H^+\right]$$

$$1.8 = \log\left(\left[H^+\right]^{-1}\right)$$

$$10^{1.8} = \left[H^+\right]^{-1}$$

$$\left[H^+\right] = \frac{1}{10^{1.8}}$$

$$= 0.0158...$$

$$\approx 0.016$$

Concentration of hydrogen ions is 0.016 mole/litre.

Question 10

a $pH = -\log[H^+]$

$pH = 0$, $[H^+] = ?$

$$0 = -\log\left[H^+\right]$$

$$0 = \log\left(\left[H^+\right]^{-1}\right)$$

$$10^{0} = \left[H^+\right]^{-1}$$

$$\left[H^+\right] = \frac{1}{10^{0}}$$

$$= 1$$

Hydrogen ion concentration for battery acid is 1 mole/litre.

b pH = 3, $[H^+]$ = ?

$$3 = -\log\left[H^+\right]$$

$$3 = \log\left(\left[H^+\right]^{-1}\right)$$

$$10^3 = \left[H^+\right]^{-1}$$

$$\left[H^+\right] = \frac{1}{10^3}$$

$$= 10^{-3}$$

$$= 0.001$$

Hydrogen ion concentration for vinegar is 10^{-3} or 0.001 mole/litre.

c pH = 8, $[H^+]$ = ?

$$8 = -\log\left[H^+\right]$$

$$8 = \log\left(\left[H^+\right]^{-1}\right)$$

$$10^8 = \left[H^+\right]^{-1}$$

$$\left[H^+\right] = \frac{1}{10^8}$$

$$= 10^{-8}$$

Hydrogen ion concentration for baking soda is 10^{-8} mole/litre.

d pH = 11, $[H^+]$ = ?

$$11 = -\log\left[H^+\right]$$

$$11 = \log\left(\left[H^+\right]^{-1}\right)$$

$$10^{11} = \left[H^+\right]^{-1}$$

$$\left[H^+\right] = \frac{1}{10^{11}}$$

$$10^{-11}$$

Hydrogen ion concentration for household ammonia is 10^{-11} mole/litre.

Question 11

a Mercury: $m_1 = -2, I_1 = 6.31$

SN1006: $m_2 = -7, I_2 = ?$

$$m_1 - m_2 = -2.5 \log10 \left(\frac{I_1}{I_2}\right)$$

$$-2-(-7) = -2.5\log_{10}\left(\frac{6.31}{I_2}\right)$$

$$5 = -2.5\log_{10}\left(\frac{6.31}{I_2}\right)$$

$$-2 = \log_{10}\left(\frac{6.31}{I_2}\right)$$

$$10^{-2} = \frac{6.31}{I_2}$$

$$I_2 = \frac{6.31}{10^{-2}}$$

$$I_2 = 631$$

b Mercury: $m_1 = -2, I_1 = 6.31$

Proxima Centauri: $m_2 = 11, I_2 = ?$

$$-2-(11) = -2.5\log_{10}\left(\frac{6.31}{I_2}\right)$$

$$-13 = -2.5\log_{10}\left(\frac{6.31}{I_2}\right)$$

$$5.2 = \log_{10}\left(\frac{6.31}{I_2}\right)$$

$$10^{5.2} = \frac{6.31}{I_2}$$

$$I_2 = \frac{6.31}{10^{5.2}}$$

$$I_2 = 0.000\ 039\ 81...$$

$$\approx 3.98 \times 10^{-5}$$

Brightness of Proxima Centauri is approximately 0.000 0398 or 3.98×10^{-5}.

c Sun: $m_1 = -27$, Moon: $m_2 = -13$

$$-27-(-13) = -2.5\log_{10}\left(\frac{I_1}{I_2}\right)$$

$$-14 = -2.5\log_{10}\left(\frac{I_1}{I_2}\right)$$

$$5.6 = \log_{10}\left(\frac{I_1}{I_2}\right)$$

$$10^{5.6} = \frac{I_1}{I_2}$$

$$\frac{I_1}{I_2} = 10^{5.6} = 398107$$

The Sun is approximately 398 107 (or about 400 000) times brighter than the Moon.

Question 12

a When $t = 5750$, $N(t) = \dfrac{N_0}{2}$

$N(t) = N_0 \times 2.72^{-kt}$

$$\frac{N_0}{2} = N_0 \times 2.72^{-k \times 5750}$$

$$0.5 = 2.72^{-5750k}$$

$$\log(0.5) = \log\left(2.72^{-5750k}\right)$$

$$\log(0.5) = -5750k \log(2.72)$$

$$-5750k = \frac{\log(0.5)}{\log(2.72)}$$

$$= -0.692709...$$

$$k = 0.000120471...$$

$$k \approx 0.0001205$$

b Skeleton loses 70% of carbon-14 atoms $\Rightarrow$ skeleton has 30% left.

$\therefore N(t) = 0.3N_0$

$$0.3N_0 = N_0 \times 2.72^{-0.0001205t}$$

$$0.3 = 2.72^{-0.0001205t}$$

$$\log(0.3) = \log\left(2.72^{-0.0001205t}\right)$$

$$\log(0.3) = -0.0001205t \log(2.72)$$

$$-0.0001205t = \frac{\log(0.3)}{\log(2.72)}$$

$$= -1.2032125...$$

$$t = 9985.16612...$$

$$t \approx 9986$$

The skeleton is approximately 9986 years old.

Question 13

a $Q(t) = Q_0 \times 2.5^{-nt}$

Initial amount occurs when $t = 0$.

$Q(t) = Q_0 \times 2.5^{-n\times 0}$

$Q(t) = Q_0 \times 2.5^{0}$

$Q(t) = Q_0$

b When $t = 1000$, $Q(t) = \frac{Q_0}{2}$

$$\frac{Q_0}{2} = Q_0 \times 2.5^{-n\times 1000}$$

$$0.5 = 2.5^{-1000n}$$

$$\log(0.5) = \log\left(2.5^{-1000n}\right)$$

$$\log(0.5) = -1000n\log(2.5)$$

$$-1000n = \frac{\log(0.5)}{\log(2.5)}$$

$$= -0.7564707...$$

$$n = 0.00075647...$$

$$n \approx 0.00076$$

c $Q_0 = 45$

$$Q(10) = 45 \times 2.5^{-0.00076\times 10}$$

$$= 45 \times 2.5^{-0.0076}$$

$$= 44.6877...$$

$$\approx 44.7$$

In 10 years there would be 44.7 kg. $\therefore$ The radioactive substance would decay.

45 – 44.7 = 0.3 kg in 10 years.

Question 14

a $P = 6.9(1.011)^t$

2011 to 2045 = 34 years, $\therefore t = 34$.

$P = 6.9(1.011)^{34}$

$P = 10.00894...$

$P \approx 10.0$

$\therefore$ Population in 2045 will be approximately 10.0 billion.

b In 2011 $t = 0$

$P = 6.9(1.011)^0$

$P = 6.9$

Hence, let $P = 6.9 \times 2 = 13.8$

$13.8 = 6.9(1.011)^t$

$2 = (1.011)^t$

$\log(2) = \log(1.011)^t$

$\log(2) = t\log(1.011)$

$$t = \frac{\log(2)}{\log(1.011)}$$

$$t = 63.3593...$$

$$t \approx 64 \text{ years}$$

(Need to round up to allow population to double.)

2011 + 64 = 2075

$\therefore$ The population will double in 2075.

Question 15

$N = 100\ (2)^{\frac{t}{15}}$

Let $N = 300$

$$300 = 100\left(2\right)^{\frac{t}{15}}$$

$$3 = \left(2\right)^{\frac{t}{15}}$$

$$\log(3) = \log\left(2\right)^{\frac{t}{15}}$$

$$\log(3) = \frac{t}{15}\log\left(2\right)$$

$$\frac{t}{15} = \frac{\log(3)}{\log\left(2\right)}$$

$$\frac{t}{15} = 1.58496...$$

$$t = 23.7744...$$

$$t \approx 24$$

There will be 300 bacteria after 24 hours.

Chapter review

Question 1

a $2^5 = 32$

$\log_2(32) = 5$

b $64^{\frac{2}{3}} = 16$

$\log_{64}(16) = \frac{2}{3}$

c $6^{-3} = \frac{1}{216}$

$\log_6\left(\frac{1}{216}\right) = -3$

Question 2

a $\log_3(81) = 4$

$3^4 = 81$

b $\log_{25}(5) = \frac{1}{2}$

$25^{\frac{1}{2}} = 5$

c $\log(0.001) = -3$

$10^{-3} = 0.001$

d $\log_8(32) = \frac{5}{3}$

$8^{\frac{5}{3}} = 32$

Question 3

a $\log_7(1)$

$= \log_7(7^0)$

$= 0\log_7(7)$

$= 0 \times 1$

$= 0$

b $\log_2(0.25)$

$= \log_2\left(\frac{1}{4}\right)$

$= \log_2\left(2^{-2}\right)$

$= -2\log_2(2)$

$= -2 \times 1$

$= -2$

c $2\log_6(6)$

$= 2 \times 1$

$= 2$

Question 4

a $\log_3(27) + \log_3(9) - \log_3(81)$

$= \log_3\left(3^3\right) + \log_3\left(3^2\right) - \log_3\left(3^4\right)$

$= 3\log_3(3) + 2\log_3(3) - 4\log_3(3)$

$= \log_3(3)$

$= 1$

b $2\log_4(7) - 2\log_4(28)$

$= \log_4\left(7^2\right) - \log_4\left(28^2\right)$

$= \log_4\left(\frac{49}{784}\right)$

$= \log_4\left(\frac{1}{16}\right)$

$= \log_4\left(4^{-2}\right)$

$= -2\log_4(4)$

$= -2\times 1$

$= -2$

c $\log_3\left(\sqrt[4]{\frac{1}{81}}\right)$

$= \log_3\left(\frac{1}{3^4}\right)^{\frac{1}{4}}$

$= \log_3\left(3^{-4}\right)^{\frac{1}{4}}$

$= \log_3\left(3^{-1}\right)$

$= -1\log_3(3)$

$= -1\times 1$

$= -1$

d $49^{\log_7 5}$

$= 7^{2\log_7 5}$

$= 7^{\log_7\left(5^2\right)}$

$= 5^2$

$= 25$

Question 5

a $\log_5 (9x^3)$

$$= \log_5(9) + \log_5(x^3)$$

$$= \log_5(9) + 3\log_5(x)$$

b $\log_7 \left(\frac{3}{z^4}\right)$

$$= \log_7(3) - \log_7(z^4)$$

$$= \log_7(3) - 4\log_7(z)$$

c $\log_7 \left[\frac{m^2(m-4)}{(m+3)^2}\right]$

$$= \log_7\left[m^2(m-4)\right] - \log_7\left[(m+3)^2\right]$$

$$= \log_7(m^2) + \log_7(m-4) - 2\log_7(m+3)$$

$$= 2\log_7(m) + \log_7(m-4) - 2\log_7(m+3)$$

d $\log_6 \left(\sqrt{\frac{x^3-y}{2xy}}\right)$

$$= \log_6\left(\left(\frac{x^3-y}{2xy}\right)^{\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left[\log_6\left(\frac{x^3-y}{2xy}\right)\right]$$

$$= \frac{1}{2}\left[\log_6(x^3-y) - \log_6(2xy)\right]$$

$$= \frac{1}{2}\left[\log_6(x^3-y) - (\log_6(2) + \log_6(x) + \log_6(y))\right]$$

$$= \frac{1}{2}\left[\log_6(x^3-y) - \log_6(2) - \log_6(x) - \log_6(y)\right]$$

Question 6

a $2\log_5(x) + 5\log_5(y)$

$$= \log_5\left(x^2\right) + \log_5\left(y^5\right)$$

$$= \log_5\left(x^2y^5\right)$$

b $2\left[\log(m) + \log(3 - m) - \log(2m^3 + 7)\right]$

$$= 2\left[\log\left(\frac{m(3-m)}{2m^3+7}\right)\right]$$

$$= \log\left[\frac{m(3-m)}{2m^3+7}\right]^2$$

c $\frac{1}{2}\log_5(17) - 3\log_5(x) - 4\log_5(y)$

$$= \log_5\left(17^{\frac{1}{2}}\right) - \log_5\left(x^3\right) - \log_5\left(y^4\right)$$

$$= \log_5\left(\frac{\sqrt{17}}{x^3y^4}\right)$$

Question 7

$\log_7(25)$

$$= \frac{\log 25}{\log 7}$$

$$= \frac{1.39794...}{0.84509...}$$

$$= 1.65417...$$

$$\approx 1.6542$$

Question 8

a $\log_m(60)$

$= \log_m(3 \times 4 \times 5)$

$= \log_m(3 \times 2^2 \times 5)$

$= \log_m(3) + \log_m(2^2) + \log_m 5$

$= \log_m(3) + 2\log_m(2) + \log_m 5$

$= 1.15 + 2 \times 0.73 + 1.68$

$= 4.29$

b $\log_m\left(\frac{8}{15}\right)$

$= \log_m\left(\frac{2^3}{3 \times 5}\right)$

$= \log_m(2^3) - \log_m(3) - \log_m(5)$

$= 3\log_m(2) - \log_m(3) - \log_m 5$

$= 3 \times 0.73 - 1.15 - 1.68$

$= -0.64$

c $\log_m\left(\sqrt{\frac{2}{9}}\right)$

$= \log_m\left(\frac{2^{\frac{1}{2}}}{3}\right)$

$= \log_m\left(2^{\frac{1}{2}}\right) - \log_m(3)$

$= \frac{1}{2}\log_m(2) - \log_m(3)$

$= \frac{1}{2} \times 0.73 - 1.15$

$= -0.785$

Question 9

a $4^{x-1} = 64$

$$4^{x-1} = 4^3$$

$$x - 1 = 3$$

$$x = 4$$

b $3^{2x-4} = 9^{2-x}$

$$3^{2x-4} = 3^{2(2-x)}$$

$$2x - 4 = 4 - 2x$$

$$4x = 8$$

$$x = 2$$

c $9^x = \sqrt{3}$

$$\left(3^2\right)^x = (3)^{\frac{1}{2}}$$

$$2x = \frac{1}{2}$$

$$x = \frac{1}{4}$$

d $9^{5-9x} = \dfrac{1}{27^{x-2}}$

$$\left(3^2\right)^{5-9x} = \left(3^{-3}\right)^{x-2}$$

$$2(5 - 9x) = -3(x - 2)$$

$$10 - 18x = -3x + 6$$

$$-15x = -4$$

$$x = \frac{4}{15}$$

Question 10

a $5^{2x} - 24 \times 5^x = 25$

$$\text{Let } a = 5^x$$

$$\left(5^x\right)^2 - 24\left(5^x\right) = 25$$

$$a^2 - 24a = 25$$

$$a^2 - 24a - 25 = 0$$

$$(a-25)(a+1) = 0$$

$$a = 25 \text{ or } a = -1$$

$$5^x = 25 \text{ or } 5^x = -1$$

$$5^x = 5^2 \text{ or no solution}$$

$$x = 2$$

b $2^{2x} - 3 \times 2^x + 2 = 0$

$$\text{Let } a = 2^x$$

$$a^2 - 3a + 2 = 0$$

$$(a-1)(a-2) = 0$$

$$a = 1 \text{ or } a = 2$$

$$2^x = 1 \text{ or } 2^x = 2$$

$$2^x = 2^0 \text{ or } 2^x = 2^1$$

$$x = 0 \text{ or } x = 1$$

Question 11

a $5^{2x-3} = 84$

$$\log\left(5^{2x-3}\right) = \log(84)$$

$$(2x-3)\log(5) = \log(84)$$

$$2x-3 = \frac{\log(84)}{\log(5)}$$

$$2x-3 = 2.75302...$$

$$2x = 5.75302...$$

$$x = 2.87651...$$

$$x \approx 2.877$$

b $7^{2x-3} = 5^{3x+1}$

$$\log\left(7^{2x-3}\right) = \log\left(5^{3x+1}\right)$$

$$(2x-3)\log(7) = (3x+1)\log(5)$$

$$\frac{(2x-3)}{(3x+1)} = \frac{\log(5)}{\log(7)}$$

$$\frac{(2x-3)}{(3x+1)} = 0.827087...$$

$$2x-3 = 0.827087...(3x+1)$$

$$2x-3 = 2.481262...x + 0.827087...$$

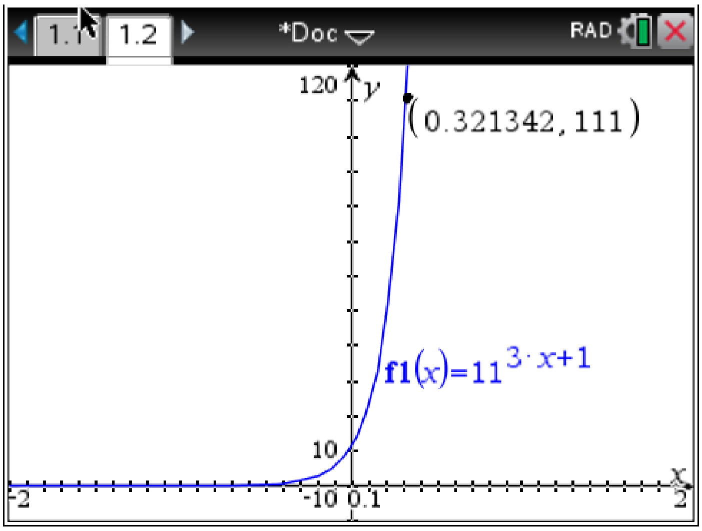
$$0.481262...x = -3.827087...$$

$$x = -7.95219...$$

$$x \approx -7.952$$

Question 12

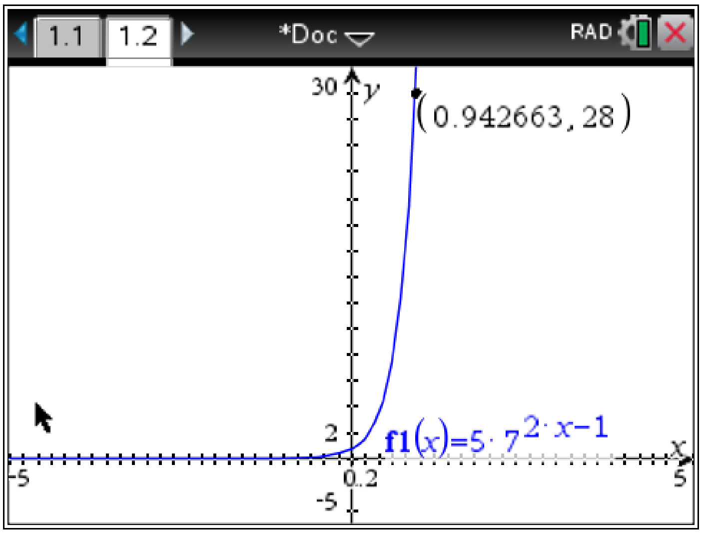
a $11^{3x+1} = 111$



$x \approx 0.321$

b $5 \times 7^{2x-1} - 28 = 0$

$5 \times 7^{2x-1} = 28$



$x \approx 0.943$

Question 13

a $y = \log_3(x) + 2$

$a = 3$, so $a > 1$

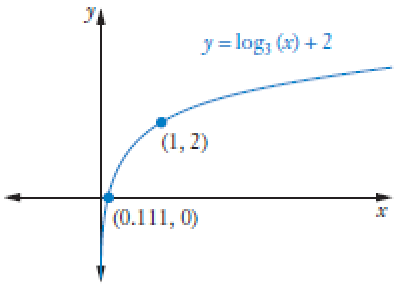
$b = 2$, $\log_3(x)$ is translated 2 units up.

$$\text{Zero} = a^{-b}$$
$$= 3^{-2}$$
$$= \frac{1}{9}$$
$$\approx 0.111$$

The graph passes through (0.111, 0).

The graph passes through (1, b).

(1, b) = (1, 2)



b $y = \log_{0.2}(x) - 2$

$a = 0.2$, so $0 < a < 1$

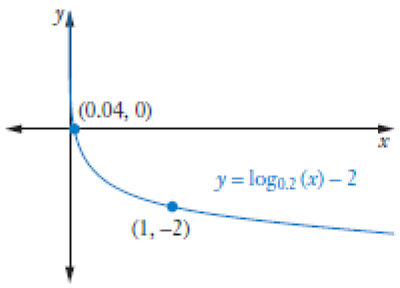
$b = -2$, $\log_{0.2}(x)$ is translated 2 units down.

$$\text{Zero} = a^{-b}$$
$$= 0.2^{-(-2)}$$
$$= 0.04$$

The graph passes through (0.04, 0).

The graph passes through (1, b).

(1, b) = (1, –2)



Question 14

a $y = \log_2 (x + 3)$

$a = 2$, so $a > 1$

$c = 3$, $\log_2(x)$ is translated 3 units to the left.

$$\begin{aligned}\text{Zero} &= 1 - c \\ &= 1 - 3 \\ &= -2\end{aligned}$$

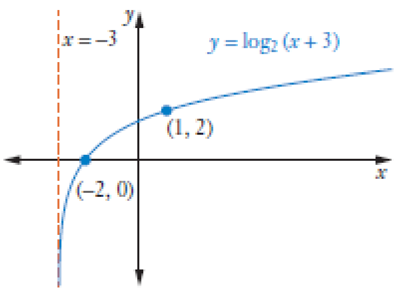
The graph passes through (–2, 0).

$2^2 = 4$, so

$$\begin{aligned}\log_2(4) &= 2 \\ x + 3 &= 2 \\ x &= 1\end{aligned}$$

The graph passes through (1, 2).

Vertical asymptote is at $x = -3$.



b $y = \log_{0.6}(x - 2)$

$a = 0.6$, so $0 < a < 1$

$c = -2$, $\log_{0.5}(x)$ is translated 2 units to the right.

$$\begin{aligned}\text{Zero} &= 1 - c\\ &= 1 - (-2)\\ &= 3\end{aligned}$$

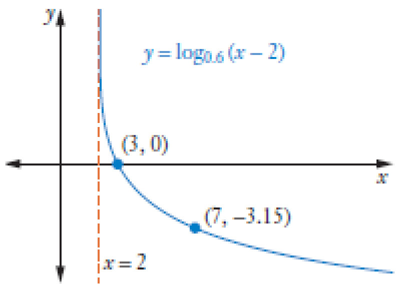
The graph passes through (3, 0).

$(0.6)^{-3.15} = 5$, so

$$\begin{aligned}\log_{0.6}(5) &= -3.15\\ x - 2 &= 5\\ x &= 7\end{aligned}$$

The graph passes through (7, –3.15).

Vertical asymptote is at $x = 2$.



Question 15

a $\log_2 (5x) = \log_2 (2x + 9)$

$5x = 2x + 9$

$3x = 9$

$x = 3$

Check: Let $x = 3$

$\log_2 (5 \times 3) = \log_2 (2 \times 3 + 9)$

$\log_2 (15) = \log_2 (15)$

$\therefore x = 3$ is a solution.

b $\log_5 (4m - 5) = \log_5 (2m - 1)$

$4m - 5 = 2m - 1$

$2m = 4$

$m = 2$

Check: Let $m = 2$

$\log_5 (4 \times 2 - 5) = \log_5 (2 \times 2 - 1)$

$\log_5 (3) = \log_5 (3)$

$\therefore m = 2$ is a solution.

c $\log_3 (16 + 2k) = \log_3 (k^2 - 4k)$

$$16 + 2k = k^2 - 4k$$

$$k^2 - 6k - 16 = 0$$

$$(k - 8)(k + 2) = 0$$

$$k = 8 \text{ or } k = -2$$

Check: Let $k = -2$

$$\log_3 (16 + 2 \times -2) = \log_3 ((-2)^2 - 4 \times (-2))$$

$$\log_3 (16 - 4) = \log_3 (4 + 8)$$

$$\log_3 (12) = \log_3 (12)$$

$\therefore k = -2$ is a solution.

Check: Let $k = 8$

$$\log_3 (16 + 2 \times 8) = \log_3 ((8)^2 - 4 \times (8))$$

$$\log_3 (16 + 16) = \log_3 (64 - 32)$$

$$\log_3 (32) = \log_3 (32)$$

$\therefore k = 8$ is a solution.

d $\log_7(2p)+\log_7(p+2)=\log_7(6)$

$$\log_7(2p)+\log_7(p+2)=\log_7(6)$$

$$\log_7(2p(p+2))=\log_7(6)$$

$$2p^2+4p=6$$

$$2p^2+4p-6=0$$

$$p^2+2p-3=0$$

$$(p+3)(p-1)=0$$

$$p=-3 \text{ or } p=1$$

Check: Let $p = -3$

$$\log_7(2\times-3)+\log_7(-3+2)=\log_7(6)$$

$$\log_7(-6)+\log_7(-1)=\log_7(6)$$

Log of negative undefined $\therefore$ $p = -3$ is not a solution.

Check: Let $p = 1$

$$\log_7(2\times1)+\log_7(1+2)=\log_7(6)$$

$$\log_7(2)+\log_7(3)=\log_7(6)$$

$$\log_7(2\times3)=\log_7(6)$$

$$\log_7(6)=\log_7(6)$$

$\therefore p = 1$ is a solution.

Question 16

a $\log_2 (5w + 6) = 4$

$$5w + 6 = 2^4 = 16$$

$$5w = 10$$

$$w = 2$$

Check: Let $w = 2$

$$\log_2 (5 \times 2 + 6) = 4$$

$$\log_2 (16) = 4$$

$$\log_2 (2^4) = 4$$

$$4 \log_2 (2) = 4$$

$$4 = 4$$

$\therefore w = 2$ is a solution.

b $\log_3(v) + \log_3(v-6) = 3$

$$\log_3(v)+\log_3(v-6)=3\log_3(3)$$

$$\log_3(v(v-6))=\log_3(3^3)$$

$$v^2-6v=27$$

$$v^2-6v-27=0$$

$$(v+3)(v-9)=0$$

$$v=-3 \text{ or } v=9$$

Check: Let $v = -3$

$$\log_3(-3)+\log_3(-3-6)=3$$

$$\log_3(-3)+\log_3(-9)=3$$

Log of negative undefined $\therefore$ $v = -3$ is not a solution.

Check: Let $v = 9$

$$\log_3(9)+\log_3(9-6)=3$$

$$\log_3(3^2)+\log_3(3)=3$$

$$2\times1+1=3$$

$$3=3$$

$\therefore v = 9$ is a solution.

c $\log_2(m+1) - \log_2(m-4) = 3$

$$\log_2(m+1) - \log_2(m-4) = 3\log_2(2)$$

$$\log_2\left(\frac{m+1}{m-4}\right) = \log_2\left(2^3\right)$$

$$\frac{m+1}{m-4} = 8$$

$$m+1 = 8(m-4)$$

$$m+1 = 8m-32$$

$$7m = 33$$

$$m = \frac{33}{7} = 4\frac{5}{7}$$

Check: Let $m = \frac{33}{7}$

$$\log_2\left(\frac{33}{7}+1\right) - \log_2\left(\frac{33}{7}-4\right) = 3$$

$$\log_2\left(\frac{40}{7} \div \frac{5}{7}\right) = 3$$

$$\log_2(8) = 3$$

$$\log_2\left(2^3\right) = 3$$

$$3\log_2(2) = 3$$

$$3 = 3$$

$\therefore m = \frac{33}{7}$ is a solution.

d $\log_{12}(x) + \log_{12}(x-1) = 1$

$$\log_{12}(x)+\log_{12}(x-1)=\log_{12}(12)$$

$$\log_{12}(x(x-1))=\log_{12}(12)$$

$$x^2-x=12$$

$$x^2-x-12=0$$

$$(x-4)(x+3)=0$$

$$x=4 \text{ or } x=-3$$

Check: Let $x = -3$

$$\log_{12}(-3)+\log_{12}(-3-1)=1$$

$$\log_{12}(-3)+\log_{12}(-4)=3$$

Log of negative undefined $\therefore$ $x = -3$ is not a solution.

Check: Let $x = 4$

$$\log_{12}(4)+\log_{12}(4-1)=1$$

$$\log_{12}(4\times3)=1$$

$$\log_{12}=1$$

$$1=1$$

$\therefore x = 4$ is a solution.

Question 17

3 t TNT $\Leftrightarrow$ M = 3.5 $\therefore 10^{1.5M} = 10^{1.5\times 3.5} = 10^{5.25}$

x t TNT $\Leftrightarrow$ M = 9.1 $\therefore 10^{1.5M} = 10^{1.5\times 9.1} = 10^{13.65}$

$$\frac{x}{3000} = \frac{10^{13.65}}{10^{5.25}} = 10^{8.4}$$

$$x = 3000\times 10^{8.4}\ \text{t}$$

= 753 565 929.5 t

= 753.565 929 5 Mt

≈ 754 Mt

Question 18

$$L = 10\log\left(\frac{I}{I_0}\right)$$

$$L = 10\log\left(\frac{100}{10^{-12}}\right)$$

$$L = 10\log\left(10^{12}\times 10^{2}\right)$$

$$L = 10\log\left(10^{14}\right)$$

$$L = 10\times 14\log\left(10\right)$$

$$L = 140$$

The loudness of the rock concert measures 140 dB.

$\therefore$ You will need ear plugs.

Question 19

$FV = PV\left(1+\frac{r}{n}\right)^{nt}$

$PV = 6500$, $r = 0.08$, $n = 12$, $t = 7$

$PV = 6500\left(1+\frac{0.08}{12}\right)^{12\times 7}$

$PV = 6500(1.00666...)^{84}$

$PV = 11358.24333...$

$PV \approx 11358.24$

$FV = PV\left(1+\frac{r}{n}\right)^{nt}$

$PV = 6500$, $r = 0.08$, $n = 365$, $t = 7$

$PV = 6500\left(1+\frac{0.08}{365}\right)^{365\times 7}$

$PV = 6500(1.000219...)^{2555}$

$PV = 11378.67296...$

$PV \approx 11378.67$

Extra money = \$11 378.67 – \$11 358.24 = \$20.43 ≈ \$20.40

Question 20

$8^b = 4^{2a+3}$

$\left(2^3\right)^b = \left(2^2\right)^{2a+3}$

$2^{3b} = 2^{4a+6}$

$3b = 4a + 6 \quad [1]$

$\log_2(b) = \log_2(a) + 3$

$\log_2(b) = \log_2(a) + 3\log_2(2)$

$\log_2(b) = \log_2(a) + \log_2\left(2^3\right)$

$\log_2(b) = \log_2(8a)$

$b = 8a \quad [2]$

Substitute [2] into [1]

$3 \times 8a = 4a + 6$

$24a = 4a + 6$

$20a = 6$

$a = \frac{3}{10}$

$b = 8 \times \frac{3}{10} = \frac{12}{5} = 2\frac{2}{5}$

Question 21

$y = \log_3 (9x + 20) - 1$

$= \log_3 9\left(x+\frac{20}{9}\right)-1$

$= \log_3(9)+\log_3\left(x+\frac{20}{9}\right)-1$

$= \log_3\left(3^2\right)+\log_3\left(x+\frac{20}{9}\right)-1$

$= 2\log_3(3)+\log_3\left(x+\frac{20}{9}\right)-1$

$= 2+\log_3\left(x+\frac{20}{9}\right)-1$

$= \log_3\left(x+\frac{20}{9}\right)+1$

$\therefore$ Vertical translation of 1 unit up and horizontal translation of $\frac{20}{9}$ units to the left.

Question 22

$m = \frac{1}{2}+2\log_9(x)$

$2m = 1+4\log_9(x)$

$2m - 1 = 4\log_9(x)$

$\frac{2m-1}{4} = \log_9(x)$

$9^{\frac{2m-1}{4}} = x$

$x = 9^{\frac{2m-1}{4}}$

Question 23

Let $\log_x(81) = z$

$\therefore x^z = 81 \quad [1]$

$m = \log_3(3x^2)$

$\therefore 3^m = 3x^2$

$3^{m-1} = x^2$

$\left(3^{m-1}\right)^{\frac{1}{2}} = \left(x^2\right)^{\frac{1}{2}}$

$3^{\left(\frac{m-1}{2}\right)} = x \quad [2]$

Substitute [2] into [1].

$$\left[3^{\left(\frac{m-1}{2}\right)}\right]^z = 81$$

$$3^{z\left(\frac{m-1}{2}\right)} = 3^4$$

$$\frac{z(m-1)}{2} = 4$$

$$z(m-1) = 8$$

$$z = \frac{8}{m-1}$$

$$\therefore \log_x(81) = \frac{8}{m-1}$$