Name: _____

TOPIC TEST

Logarithmic functions

- Time allowed: 45 minutes
- Part A: 10 multiple-choice questions (10 marks)
- Part B: 9 free-response questions (40 marks)
- Total: 50 marks

Part A

10 multiple-choice questions
1 mark each: 10 marks
Circle the correct answer.

1 $\log_2\left(\frac{1}{8}\right) =$

A -3 **B** 3

C $\frac{1}{3}$ **D** 2

2 $\log_{10}(1) =$

A 0 **B** 1

C 2 **D** 10

3 $\log_a(\sqrt{a}) =$

A a **B** $\frac{1}{2}$

C 1 **D** 2

4 Simplify $\log_4(x) - \log_4(x+4)$.

A $\log_4\left(\frac{x+4}{x}\right)$ **B** $\log_4\left(\frac{x}{x-4}\right)$

C $\log_4\left(\frac{x}{x+4}\right)$ **D** -1

5 If $\log_m(12) + \log_m(x) = 0$, then $x =$

A -12 **B** 0

C $\frac{1}{12}$ **D** 1

6 Solve $\log_4(4x) = 3$.

A $x = 64$ **B** $x = 16$

C $x = 12$ **D** $x = 3$

7 Solve $3\log_2(2x-1) = -9$.

A $x = \frac{9}{16}$ **B** $x = \frac{7}{16}$

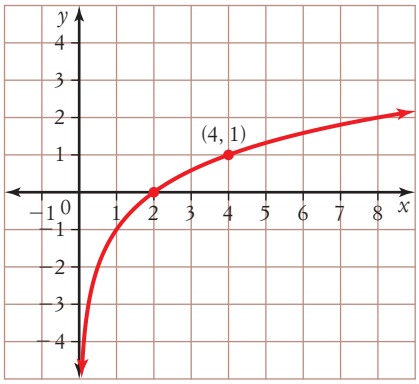
C $x = -\frac{5}{2}$ **D** $x = \frac{9}{2}$

8 Which function's graph is the graph of $y = \log_a(x)$ translated 3 units upwards?

A $y = \log_a(x+3)$ **B** $y = \log_a(x-3)$

C $y = \log_a(x) - 3$ **D** $y = \log_a(x) + 3$

9 What is the equation of this logarithmic graph?



A $y = \log_2(x-1)$ **B** $y = \log_2(x) - 1$

C $y = \log_2(x) - 2$ **D** $y = \log_2(x-2)$

10 The formula for pH is $pH = -\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions, given in mole/litre. If an orange juice has a pH of 3.6, what is the concentration of hydrogen ions?

A −0.5562 **B** 1.7976

C 0.5563 **D** 0.0003

Part B

9 free-response questions
40 marks
Show your working where appropriate.

11 Write $3^4 = 81$ as a logarithmic statement.

[1 mark]

12 Evaluate $\log_7 (35)$ correct to 4 decimal places.

[2 marks]

13 Expand $\log_a (xy^2)$ in terms of $\log_a (x)$ and $\log_a (y)$.

[2 marks]

14 Given that $\log_m (2) = 0.73$, $\log_m (3) = 1.15$ and $\log_m (5) = 1.68$, use the properties of logarithms to evaluate $\log_m\left(\sqrt{\frac{2}{15}}\right)$ correct to 2 decimal places.

[4 marks]

15 Solve each equation.

a $\log_2 (x) = -5$

b $2 \log_3 (3x - 2) = 4$

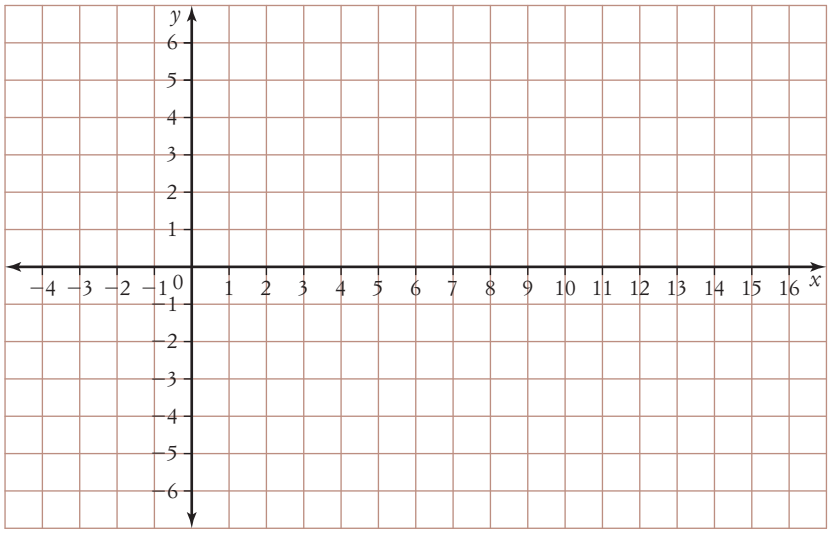
c $\log(x) + \log(x+3) = \log(5x+8)$

d $81^{2x-1} = 27$

e $5^{3x-4} = 21$, correct to 3 decimal places.

[12 marks]

16 On the same number plane below, sketch the graphs of $y = \log_4(x)$, $y = \log_4(x) + 2$ and $y = \log_4(x+2)$, labelling important features.



[6 marks]

17 The population of a city can be modelled using the formula $P = 4.6(1.08)^t$, where t is the number of years after 2017 and P is the population in millions.

a Use the formula to calculate the city's population in the year 2037, to the nearest 100 million.

b In what year will the population be 10 times what it was in 2017?

[4 marks]

18 a If you deposit \$6000 into an account paying 4% annual interest compounded monthly, how long will it be until there is \$9000 in the account?

b At 3.2% annual interest compounded quarterly, how long will it take to double your money?

[5 marks]

19 One of the Richter formulas used to measure the magnitude, M, of an earthquake is:

$$M(x) = \log\left(\frac{x}{x_0}\right)$$

where x is the measure of the amplitude of the earthquake wave and x_0 is the amplitude of the standard wave.

a Find the magnitude of an earthquake correct to 2 decimal places if $x = 50\,000\,x_0$.

b How many times more intense than the reference intensity, x_0, is an earthquake with a magnitude of 5.2?

[4 marks]

This is the end of the test.
Use the rest of this page for extra working space.

Answers

Part A

1 A	**2** A	**3** B	**4** C	**5** C
6 B	**7** A	**8** D	**9** B	**10** D

Part B

11 $\log_3 81 = 4$

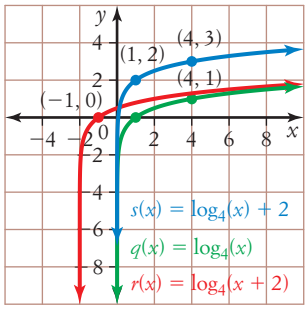
12 1.8271

13 $\log_a x + 2\log_a y$

14 −1.05

15 **a** $x = \frac{1}{32}$ **b** $x = \frac{11}{3}$ **c** $x = -2, 4$ **d** $x = \frac{7}{8}$ **e** $x = 1.964$

16



17 **a** 21.4 million **b** year 2047

18 **a** 121.8 months (just after 10 years) **b** 87 quarters (21.75 years)

19 **a** approx. 4.7 **b** approx. 3 times more: $158\,490\,x_0$